CH1. 물리학과 측정

p4 § 길이, 질량, 시간의 표준

	SI	CGS
길이 :	m	cm
질량 :	kg	g
시간 :	sec	sec

접두어

10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P

Ex) 3 nm = 3×10^{-9} (m)

6 Gm = 6×10^{9} (m)

p9

3 차원의 분석

$$A = B$$

A의 차원 = B의 차원

Ex) $x = \frac{1}{2} a t^2$

$$[x] = m$$

$$\left[\frac{1}{2} a t^2\right] = [a]\,[t]^2 = \frac{m}{sec^2}\, sec^2 = m$$ ※

p10

(예제 1.1)

$$v = a t$$

v: 속력

a: 가속도

t: 시간

$$[v] = \frac{m}{sec}$$

$$[at] = [a]\,[t] = \frac{m}{sec^2} \cdot sec = \frac{m}{sec}$$

$\Rightarrow \quad [v] = [at]$ ok ※

p10

(예제1.2)

$$a = k\, r^{n} v^{m}$$

$$[a] = \frac{m}{sec^2}$$

$$[r^n v^m] = [r]^n\,[v]^m$$

$$= m^n \left(\frac{m}{sec}\right)^m$$

$$= \frac{m^{n+m}}{sec^{m}}$$

$m = 2$. $n = -1$

$$\Rightarrow a = k\,\frac{v^2}{r}$$

※

p11

※ 단위의 환산

1 mile = 1609 m = 1.609 km

1 ft = 30.48 cm = 0.3048 m

1 m = 3.281 ft = 39.37 in

1 in = 2.54 cm = 0.0254 m

(Ex)

$$15\ \text{in} = 15\ \text{in}\ \frac{2.54\ \text{cm}}{1\ \text{in}} = 15 \times 2.54\ \text{cm} = 38.1\ \text{cm}$$

(예2)

$$100\ \text{mi} = 100\ \text{mi}\ \frac{1.609\ \text{km}}{1\ \text{mi}} = 160.9\ (\text{km})$$

(예제1.3)

38 m/sec

$$= 38\ \frac{\cancel{\text{m}}}{\cancel{\text{sec}}}\ \frac{1\ \text{mile}}{1609\ \cancel{\text{m}}}\ \frac{3600\ \cancel{\text{sec}}}{1\ \text{h}}$$

$$= \frac{38 \times 3600}{1609}\ (\text{mile/h})$$

$$= 85\ (\text{mile/h})$$

속도위반 !! ※

제2. 일차원에서의 운동

p18

8. 위치, 속도 그리고 속력

$x = x(t)$: 위치

$v_{x,avg} = \frac{\Delta x}{\Delta t}$: 평균속도 ($\Delta x = x_f - x_i$: 변위)

x_i : 처음위치

x_f : 마지막위치

$v_{avg} = \frac{d}{\Delta t}$: 평균속력 (d : 움직인 거리)

(예제 2.1)

$t_A = 0$	$x_A = 30$ cm	
		22 cm
$t_B = 10$ sec	$x_B = 52$ cm	
		14 cm
$t_C = 20$ sec	$x_C = 38$ cm	
		38 cm
$t_d = 30$ sec	$x_d = 0$ cm	
		37 cm
$t_e = 40$ sec	$x_e = -37$ cm	
		16 cm
$t_f = 50$ sec	$x_f = -53$ cm	
		127 cm

변위 $= x_f - x_A = (-53) - (30) = -83$ (cm)

평균속도 $= \frac{\Delta x}{\Delta t} = \frac{-83 \text{ cm}}{50 \text{ sec}} = -1.66$ (cm/sec)

$d = 127$ cm

평균속력 $= \frac{127 \text{ cm}}{50 \text{ sec}} = 2.54$ (cm/sec) //

③ 순간속도 와 속력

순간속도 $v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

순간속력 $= |v_x|$

p23

(예제2.2)

$x = -4t + 2t^2$

(A)

$x(1) - x(0) = (-2) - 0 = -2$ (m)

$x(3) - x(1) = 6 - (-2) = 8$ (m)

(B) $v_1 = \frac{-2\,(m)}{1\,sec} = -2$ (m/sec)

$v_2 = \frac{8\ m}{2\,(sec)} = 4$ (m/sec)

(C) $v(t) = \frac{dx}{dt} = -4 + 4t$

$v(2.5) = -4 + 4 \times 2.5 = 6$ (m/sec) ※

p24

5. 등속운동 하는 입자

$$v_x = \frac{dx}{dt} = \text{const}$$

$$\Rightarrow \quad dx = v_x \, dt$$

$$\Rightarrow \quad \int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} v_x \, dt = v_x \int_{t_i}^{t_f} dt$$

$$\Rightarrow \quad x_f - x_i = v_x t \qquad (t = t_f - t_i)$$

$$\Rightarrow \quad \underline{x_f = x_i + v_x t}$$

p26

(예제2.3)

(A) $v = \dfrac{20\ \text{cm}}{4\ \text{sec}} = 5\ (\text{cm/sec})$

(B) $x_f = x_i + v_x t$

$x_i = 0, \quad v_x = 5\ (\text{cm/sec}), \quad t = 10\ (\text{sec})$

$x_f = 50\ (\text{cm}).$ //

p27

§ 가속도

평균 가속도 $a_{x,avg} = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$

순간 가속도 $a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$

* 물체의 속도와 가속도가 같은 방향일때 물체의 속력은 증가한다.
반면에 물체의 속도와 가속도가 반대일때 물체의 속력은 감소한다

p30
(예제 2.5)

$v_x = 40 - 5t^2$ (m/sec)

(A) $a_{x,avg} = \frac{v_x(2) - v_x(0)}{2.0 - 0} = \frac{20 - 40 \text{ (m/sec)}}{2 \text{ (sec)}} = -10 \text{ (m/sec}^2)$

(B) $a_x = \frac{dv_x}{dt} = -10t$

$a_x(t = 2\text{ sec}) = -20 \text{ (m/sec}^2)$

3 등가속도 운동을 하는 입자

$$a_x = \frac{dv_x}{dt} = \text{const}$$

$$\Rightarrow \quad dv_x = a_x \, dt$$

$$\Rightarrow \quad \int_{v_{x,i}}^{v_{x,f}} dv_x = a_x \int dt$$

$$\Rightarrow \quad \underline{v_{x,f} = v_{x,i} + a_x t} \quad — ①$$

$$v_{xf} = \frac{dx}{dt}$$

$$\Rightarrow \quad \int_{x_i}^{x_f} dx = \int v_{x,f} \, dt = \int (v_{x,i} + a_x t) \, dt$$

$$\Rightarrow \quad x_f - x_i = v_{x,i} t + \frac{1}{2} a_x t^2$$

$$\Rightarrow \quad \underline{x_f = x_i + v_{x,i} t + \frac{1}{2} a_x t^2} \quad — ②$$

From Eq. ②

$$x_f - x_i = v_{x,i} t + \frac{1}{2} a_x t^2 \qquad \left(\Leftarrow t = \frac{v_{x,f} - v_{x,i}}{a_x} \right)$$

$$= v_{x,i} \frac{v_{x,f} - v_{x,i}}{a_x} + \frac{1}{2} a_x \frac{(v_{x,f} - v_{x,i})^2}{a_x^2}$$

$$= \frac{v_{x,i}(v_{x,f} - v_{x,i})}{a_x} + \frac{(v_{x,f} - v_{x,i})^2}{2a_x}$$

$$= \frac{(v_{x,f} - v_{x,i})^2 + 2v_{x,i}(v_{x,f} - v_{x,i})}{2a_x}$$

$$= \frac{v_{x,f}^2 - v_{x,i}^2}{2a_x}$$

$$\Rightarrow \quad v_{x,f}^2 - v_{x,i}^2 = 2a_x(x_f - x_i) \quad \text{— ④}$$

From ②

$$v_{x,avg} \equiv \frac{x_f - x_i}{t}$$

$$= \frac{v_{x,i}t + \frac{1}{2}a_x t^2}{t}$$

$$= v_{x,i} + \frac{1}{2}a_x t \qquad (a_x t = v_{x,f} - v_{x,i} \text{ From Eq. ①})$$

$$= v_{x,i} + \frac{1}{2}(v_{x,f} - v_{x,i})$$

$$= \frac{v_{x,f} + v_{x,i}}{2}$$

$$\Rightarrow \quad v_{x,avg} = \frac{v_{x,f} + v_{x,i}}{2} \quad \text{— ⑤}$$

· Summary

$$v_{x,f} = v_{x,i} + a_x t$$

$$x_f = x_i + v_{x,i}t + \frac{1}{2}a_x t^2$$

$$v_{x,f}^2 - v_{x,i}^2 = 2a_x(x_f - x_i)$$

$$v_{x,avg} = \frac{1}{2}(v_{x,i} + v_{x,f})$$

p24

(예제 2.6)

S_1: 자동차가 움직인 거리

$$S_1 = 45t + 45$$

S_2: 순찰차가 움직인 거리

$$S_2 = \frac{1}{2}at^2 = \frac{1}{2}\cdot 3t^2 = \frac{3}{2}t^2$$

$$S_1 = S_2$$

$$\Rightarrow 45t + 45 = \frac{3}{2}t^2$$

$$\Rightarrow t^2 - 30t - 30 = 0$$

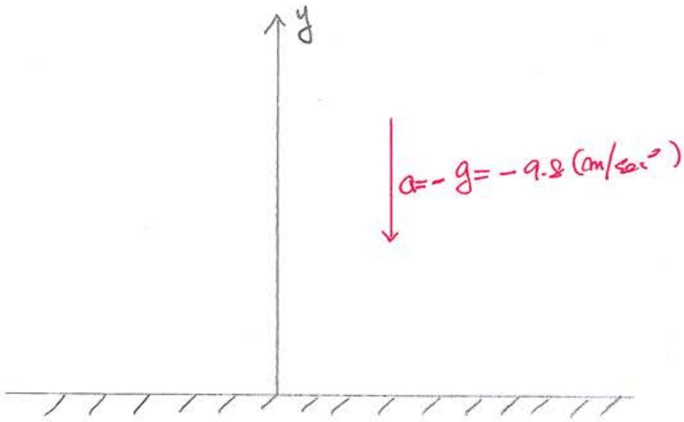
$$t = 30.9687 \text{ (sec)}$$

※

p25 ※ 자유낙하운동

자유낙하운동 = 등가속도 운동

중력가속도 $g = 9.8\ (m/sec^2)$ (지구방향)



$$\boxed{\begin{aligned} v_f &= v_i - gt \\ y_f &= y_i + v_i t - \frac{1}{2} g t^2 \\ v_f^2 - v_i^2 &= -2g(y_f - y_i) \\ v_{avg} &= \frac{1}{2}(v_i + v_f) \end{aligned}}$$

p37

(예제2.7)

(A) $v_f = v_i - gt$

$$\Rightarrow t = \frac{v_i - v_f}{g}$$

$v_i = 20\,(m/sec)$, $v_f = 0$, $g = 9.8\,(m/sec^2)$

$$\Rightarrow t = \frac{20 - 0}{9.8} = 2.04\,(sec)$$

(B) $v_f^2 - v_i^2 = -2g\,(y_f - y_i)$

$$\Rightarrow y_f = y_i - \frac{v_f^2 - v_i^2}{2g}$$

$y_i = 50\,(m)$, $v_f = 0$, $v_i = 20\,(m/sec)$, $g = 9.8\,(m/sec^2)$

$$\Rightarrow y_f = 50 - \frac{0 - (20)^2}{2 \times 9.8} = 70.4\,(m)$$ 땅 에서의 높이

(C) $v_f^2 - v_i^2 = -2g\,(y_f - y_i)$

$v_i = 20\,(m/sec)$, $y_i = y_f = 50\,(m)$

$$\Rightarrow v_f = \pm v_i$$

$$\Rightarrow v_f = -20\,(m/sec)$$

(D) $v_f = v_i - gt = 20\,(m/sec) - 9.8 \times 5 = -29\,(m/sec)$

$y_f = y_i + v_i t - \frac{1}{2}gt^2$

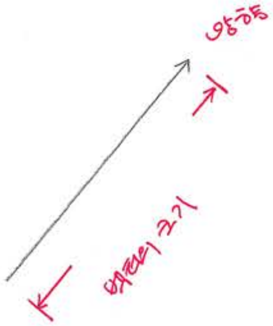
$= 50 + 20 \times 5 - \frac{1}{2} \times 9.8 \times 25$

$= 27.5\,(m)$ 땅에서의 높이

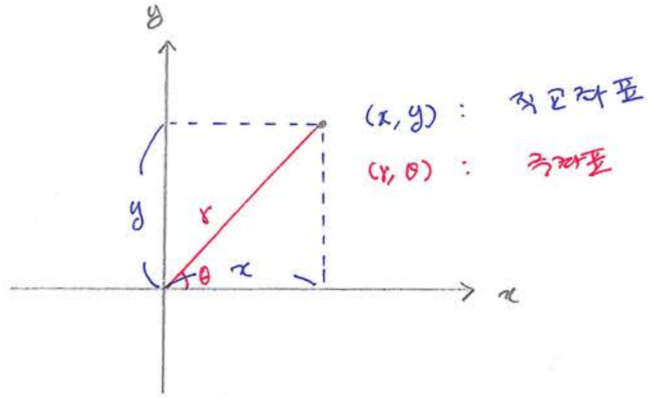
CH. 3 Vector

스칼라 (Scalar) : 크기만 있는 양 (Ex) 속력

벡터 (vector) : 크기와 방향이 있는 양 (Ex) 속도, 가속도, 힘



§ 좌표계



$$x = r\cos\theta \qquad r = \sqrt{x^2+y^2}$$

$$y = r\sin\theta \qquad \theta = \tan^{-1}\frac{y}{x}$$

p43

(예제 3.1)

$$(x, y) = (-3.5, -2.5)$$

$$r = \sqrt{(-3.5)^2 + (-2.5)^2} = 4.3 \ (m)$$

$$\tan\theta = \frac{y}{x} = \frac{2.5}{3.5} = \frac{5}{7}$$

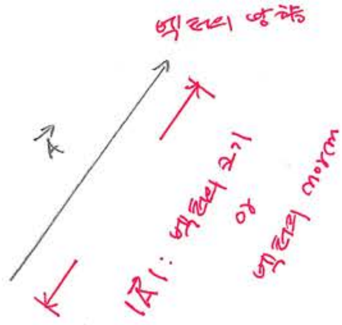
$$\theta = \tan^{-1}\left(\frac{5}{7}\right) = 215.5^\circ$$

※

p43 § 벡터량 & 스칼라량

스칼라 (scalar) : 크기만 갖는 양　　Ex) 질량

벡터 (Vector) : 크기와 방향이 있는 양　　Ex) 속도, 가속도



* $|\vec{A}|$: 스칼라

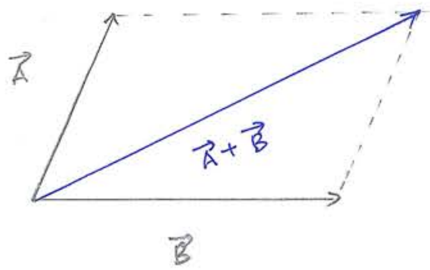
p44 § 벡터의 성질

(i) 벡터의 동등성

$\vec{A} = \vec{B}$ means

$$\left(\begin{array}{l} \vec{A}\text{의 방향} = \vec{B}\text{의 방향} \\ |\vec{A}| = |\vec{B}| \end{array} \right)$$

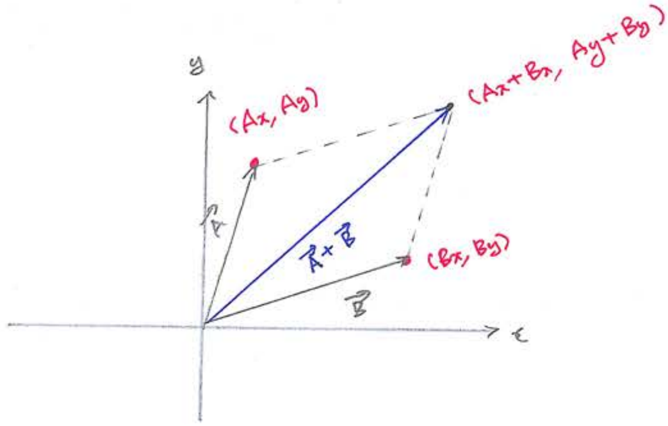
(ii) 벡터의 덧셈



$$\vec{A}+\vec{B} = \vec{B}+\vec{A}$$

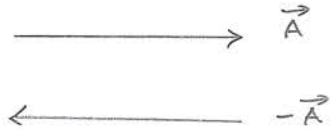
$$\vec{A}+(\vec{B}+\vec{C}) = (\vec{A}+\vec{B})+\vec{C}$$

* 좌표계를 도입하면 "벡터의 덧셈"을 쉽게 할 수 있다.

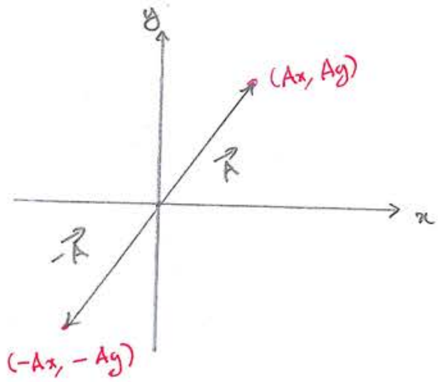


(iii) 음의 Vector

$-\vec{A}$: $\vec{A}$와 크기가 같고 방향이 반대인 Vector

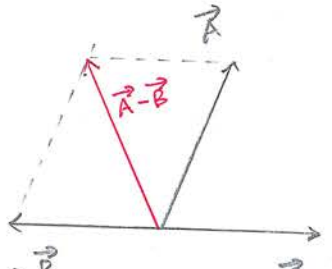


좌표계

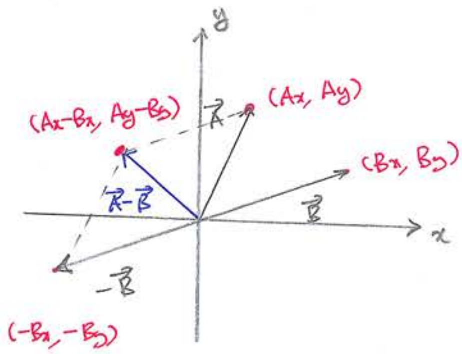


(iv) 벡터의 뺄셈

$\vec{A} - \vec{B} \equiv \vec{A} + (-\vec{B})$



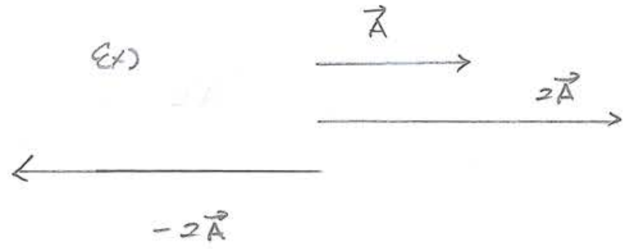
좌표계



(v) 스칼라 × Vector

$m\vec{A}$: $\vec{A}$ 와 같은 방향이고 크기가 m 배 인 vector

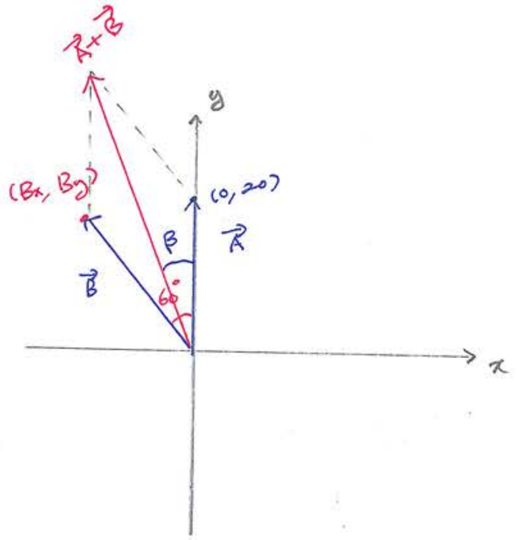
Ex)



※

p47

(예제3.2)



$$B_x = -35 \sin(60°) = -\frac{35}{2}\sqrt{3}$$

$$B_y = 35 \cos(60) = \frac{35}{2}$$

$$\vec{A}+\vec{B} = \left(-\frac{35}{2}\sqrt{3},\ 20+\frac{35}{2}\right) = (-30.3,\ 37.5)$$

$$|\vec{A}+\vec{B}| = \sqrt{(-30.3)^2+(37.5)^2} = 48.2 \text{ (km)}$$

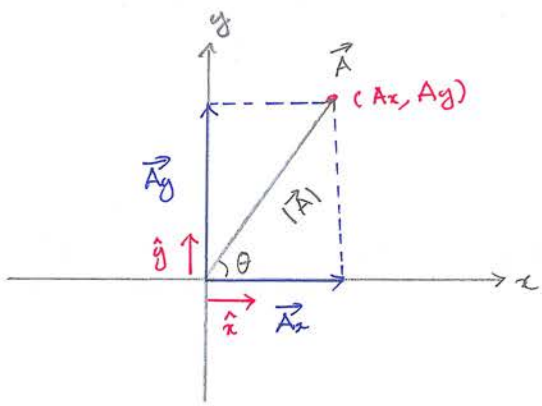
$$\tan\beta = \frac{30.3}{37.5}$$

$$\beta = 38.9°$$

"자동차 변위는 48.2 (km) 이고 방향은 북서쪽 38.9° 이다"

※

p48 ※ 벡터의 성분과 단위벡터



(A_x, A_y) : $\vec{A}$의 성분

$A_x = |\vec{A}| \cos\theta$

$A_y = |\vec{A}| \sin\theta$

$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

$\theta = \tan^{-1} \frac{A_y}{A_x}$

* 벡터 분해와 단위 벡터

$\vec{A} = \vec{A_x} + \vec{A_y}$

$|\vec{A_x}| = A_x$, $\vec{A_x}$의 방향 : x축 방향

$|\vec{A_y}| = A_y$, $\vec{A_y}$의 방향 : y축 방향

단위벡터 : 크기가 1인 vector

$\hat{x}$: x축 방향의 단위 벡터

$\hat{y}$: y축 방향의 단위 벡터

$\Rightarrow \quad \vec{A_x} = A_x \hat{x}$

$\vec{A_y} = A_y \hat{y}$

$\Rightarrow \quad \vec{A} = A_x \hat{x} + A_y \hat{y}$

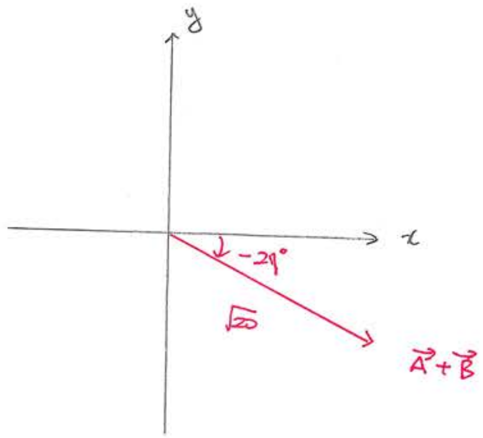
벡터의 분해

p51

(예제3.3)

$$\vec{A} = 2\hat{x} + 2\hat{y}$$

$$\vec{B} = 2\hat{x} - 4\hat{y}$$



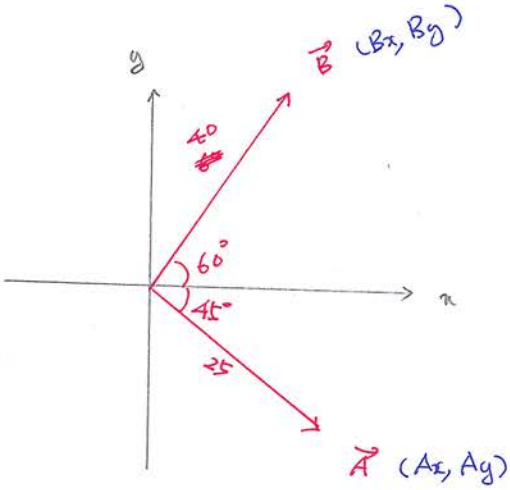
$$\vec{A}+\vec{B} = 4\hat{x} - 2\hat{y}$$

$$|\vec{A}+\vec{B}| = \sqrt{4^2+(-2)^2} = \sqrt{20}$$

$$\tan\theta = \frac{-2}{4} = -\frac{1}{2}$$

$$\theta = -27^\circ$$

p52
(예제3.4)



(A)

$$A_x = 25\cos 45^\circ = \frac{25}{\sqrt{2}} = 17.7\ (\mathrm{km})$$

$$A_y = -25\sin 45^\circ = -\frac{25}{\sqrt{2}} = -17.7\ (\mathrm{km})$$

$$\vec{A} = 17.7\,\hat{x} - 17.7\,\hat{y}$$

$$B_x = 40\cos 60^\circ = 20\ (\mathrm{km})$$

$$B_y = 40\sin 60^\circ = 40\cdot\frac{\sqrt{3}}{2} = 34.6\ (\mathrm{km})$$

$$\vec{B} = 20\,\hat{x} + 34.6\,\hat{y}$$

(B) $\vec{R} = \vec{A}+\vec{B} = 37.7\,\hat{x} + 16.9\,\hat{y}$

Ch.4 이차원 운동

§ 위치, 속도, 가속도 벡터

$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$: 변위벡터

$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$: 평균속도

$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$: 순간속도

$v = |\vec{v}|$: 순간 속력

$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$: 평균가속도 $(\Delta\vec{v} = \vec{v}_f - \vec{v}_i)$

$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$: 순간 가속도

가속도를 만드는 요인

1. 속도의 크기의 변화
2. 속도의 방향의 변화 (Ex: 등속 원운동)
3. 속도의 크기와 방향의 변화

p78
(퀴즈 4.1)

p78 ※ 등가속도 이차원 운동

$$\vec{r} = x\,\hat{x} + y\,\hat{y}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{x} + \frac{dy}{dt}\hat{y} = v_x\,\hat{x} + v_y\,\hat{y}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{x} + \frac{dv_y}{dt}\hat{y} = a_x\hat{x} + a_y\hat{y} = \text{const}$$

$$\frac{dv_x}{dt} = a_x = \text{const}$$

$$\Rightarrow \int_{v_{x,i}}^{v_{x,f}} dv_x = a_x t$$

$$\Rightarrow \quad v_{x,f} = v_{x,i} + a_x t$$

$$v_{y,f} = v_{y,i} + a_y t$$

$$\Rightarrow \quad \vec{v}_f = \vec{v}_i + \vec{a}t \qquad \Leftrightarrow \quad v_f = v_i + at$$

$$\Rightarrow \quad v_{x,f} = v_{x,i} + a_x t = \frac{dx}{dt}$$

$$\int_{x_i}^{x_f} dx = v_{x,i}t + \frac{1}{2}a_x t^2$$

$$x_f = x_i + v_{x,i}t + \frac{1}{2}a_x t^2$$

$$y_f = y_i + v_{y,i}t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad \vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \qquad \Leftrightarrow \quad x_f = x_i + v_i t + \frac{1}{2}at^2$$

P61
(예제 4.1)

$v_{x,i} = 20$ (m/sec) , $v_{y,i} = -15$ (m/sec) ,

$a_x = 4$ (m/sec^2) , $a_y = 0$, $x_i = y_i = 0$

(A) $v_{x,f} = v_{x,i} + a_x t = 20 + 4t$ (m/sec)

$v_{y,f} = v_{y,i} + a_y t = -15$ (m/sec)

$\vec{v}_f = (20+4t)\,\hat{x} - 15\,\hat{y}$

(B) $\vec{v}_f(t=5) = 40\,\hat{x} - 15\,\hat{y}$ (m/sec)

θ : x축과 $\vec{v}_f$ 사이의 각도

$\theta = \tan^{-1} \frac{-15}{40} = -21^\circ$

$v_f(t=5) = \sqrt{40^2 + (-15)^2} = 43$ (m/sec)

(C) $x_f = x_i + v_{x,i} t + \frac{1}{2} a_x t^2 = 20t + 2t^2$

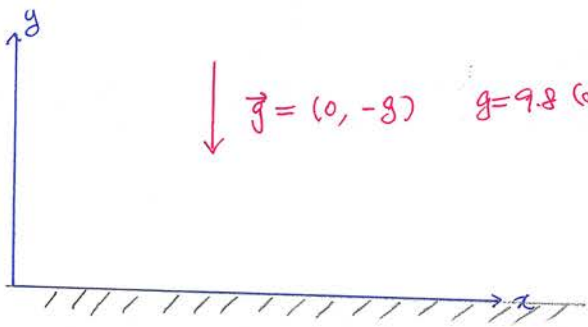
$y_f = y_i + v_{y,i} t + \frac{1}{2} a_y t^2 = -15t$

$\vec{r}_f = x_f\,\hat{x} + y_f\,\hat{y} = (20t + 2t^2)\,\hat{x} - 15t\,\hat{y}$ ※

p62 포물체 운동

$\vec{a} = \vec{g}$ (중력 가속도)

$$\boxed{\begin{aligned}\vec{v}_f &= \vec{v}_i + \vec{g}t \\ \vec{r}_f &= \vec{r}_i + \vec{v}_i t + \tfrac{1}{2}\vec{g}t^2\end{aligned}}$$



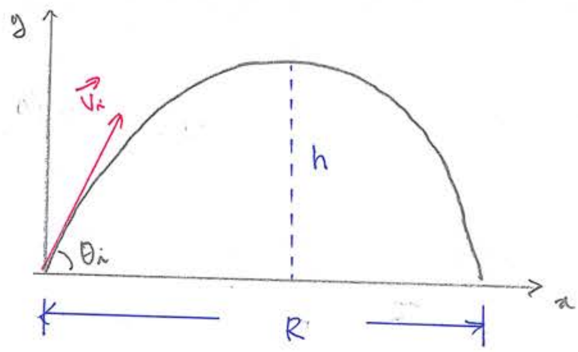
$v_{x,f} = v_{x,i}$ — ①

$v_{y,f} = v_{y,i} - gt$ — ②

$x_f = x_i + v_{x,i}t$ — ③

$y_f = y_i + v_{y,i}t - \frac{1}{2}gt^2$ — ④

* 최대높이 와 수평 도달거리



$x_i = y_i = 0 \quad v_{x,i} = v_i \cos\theta_i, \quad v_{y,i} = v_i \sin\theta_i$

최대높이 ($v_{yf} = 0$)

From ④

$$t = \frac{v_{y,i}}{g} = \frac{v_i \sin\theta_i}{g} \quad - ⑤$$

⑤ → ④

$$h = v_{y,i} \frac{v_i \sin\theta_i}{g} - \frac{1}{2} g \frac{v_i^2 \sin^2\theta_i}{g^2} = \frac{v_i^2 \sin^2\theta_i}{2g}$$

$$h = \frac{v_i^2 \sin^2\theta_i}{2g} \quad - ⑥$$

수평도달거리 ($y_f = 0$)

From ④

$$v_i \sin\theta_i t - \frac{1}{2} g t^2 = 0$$

$$t = \frac{2v_i \sin\theta_i}{g} \quad - ⑦$$

⑦ → ③

$$R = v_i \cos\theta_i \frac{2v_i \sin\theta_i}{g} = \frac{v_i^2 \sin 2\theta_i}{g}$$

$$R = \frac{v_i^2 \sin 2\theta_i}{g}$$

note) $R_{max} = \frac{v_i^2}{g}$ when $\theta_i = 45^\circ$

(퀴즈 4.3)

$$T = \frac{2v_i}{g} \sin\theta_i \quad , \quad v_i = 50 \ (m/sec)$$

15° : 2.64 (sec)

30° : 5.10 (sec)

45° : 7.21 (sec)

60° : 8.84 (sec)

75° : 9.86 (sec)

※

p65

(예제4.2)

$$\theta_i = 20^\circ, \quad v_i = 11 \ (m/sec)$$

(A) $$R = \frac{v_i^2 \sin 2\theta_i}{g} = \frac{11^2}{9.8} \sin 40^\circ = 7.94 \ (m)$$

(B) $$h = \frac{v_i^2 \sin^2\theta_i}{2g} = \frac{11^2}{2 \times 98} \sin^2 20^\circ = 0.722 \ (m) \ *$$

p65

(예제4.3)

* $x = x_T$ 에서의 총알의 높이

From Eq. ③

$$t = \frac{x_T}{v_{xi}} = \frac{x_T}{v_i \cos\theta_i} \quad — \quad (1)$$

(1) → ④

$$h_{총알} = v_i \sin\theta_i \frac{x_T}{v_i \cos\theta_i} - \frac{1}{2} g \frac{x_T^2}{v_i^2 \cos^2\theta_i}$$

$$= x_T \tan\theta_i - \frac{g x_T^2}{2 v_i^2 \cos^2\theta_i} \quad (2)$$

표적: $y_i = x_T \tan\theta_i, \quad v_0 = 0$

시간 t 에서 표적의 높이

$$h_{표적} = y_i + v_0 t - \frac{1}{2} g t^2$$

$$= x_T \tan\theta_i - \frac{1}{2} g \frac{x_T^2}{v_i^2 \cos^2\theta_i}$$

$$= x_T \tan\theta_i - \frac{g x_T^2}{2 v_i^2 \cos^2\theta_i} \quad (3)$$

$$h_{총알} = h_{표적} \quad *$$

(예제4.4)

$v_{x,i} = 25\,(m/sec)$, $v_{y,i} = 0$, $x_i = y_i = 0$

$x_f = 25t$

$y_f = -\frac{1}{2}gt^2 = -\frac{9.8}{2}t^2$

$\cos 35^\circ = \frac{x_f}{d} = \frac{25t}{d}$ — ①

$\sin 35^\circ = \frac{-y_f}{d} = \frac{9.8}{2d}t^2$ — ②

From ①

$t = \frac{d}{25}\cos 35^\circ$ — ③

③ → ②

$\sin 35^\circ = \frac{9.8}{2d}\frac{d^2}{25^2}\cos^2 35^\circ = \frac{9.8}{2\times 25^2}(\cos^2 35^\circ)\,d$

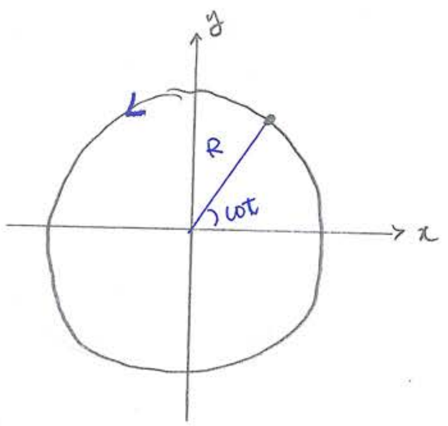
$\Rightarrow$ $d = \frac{2\times 25^2}{9.8}\frac{\sin 35^\circ}{\cos^2 35^\circ} = 109$ m

*

§ 등속 원운동

등속 원운동 : 일정한 속력으로 하는 원운동

⇒ 속도의 방향이 변화하므로 가속도가 존재한다.



$x = R\cos\omega t \quad , \quad y = R\sin\omega t$

$\vec{r} = R\cos\omega t\,\hat{x} + R\sin\omega t\,\hat{y} \qquad |\vec{r}| = R$

$\vec{v} = \frac{d\vec{r}}{dt} = -R\omega\sin\omega t\,\hat{x} + R\omega\cos\omega t\,\hat{y} \qquad |\vec{v}| = v = R\omega$

$\vec{a} = -R\omega^2\cos\omega t\,\hat{x} - R\omega^2\sin\omega t\,\hat{y} = -\omega^2\vec{r} \qquad |\vec{a}| = a = R\omega^2$

$$\boxed{a = R\omega^2 = R\frac{v^2}{R^2} = \frac{v^2}{R}}$$

방향: 원의 중심

구심 가속도

주기:

$T = \frac{2\pi R}{v}$ (주기)

p70

(예제 4.5)

지구와 태양 사이의 거리 = $r = 1.496 \times 10^{11}$ (m)

$$a = \frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$= \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

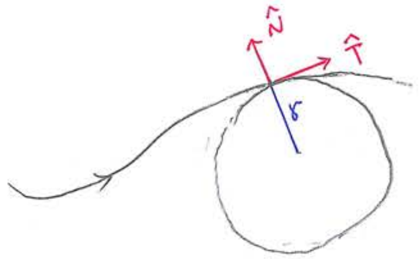
$$= \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2 \cdot 1.496\times 10^{11}\ (\text{m})}{365^2 \times 24^2 \times 3600^2\ (\text{sec}^2)}$$

$$= 5.93 \times 10^{-3} \quad (\text{m}/\text{sec}^2)$$

p70 ◎ 접선 및 지름 가속도

속도의 크기와 방향이 모두 변화하는 경우의 가속도 :



$$\vec{a} = \frac{dv}{dt}\hat{T} - \frac{v^2}{r}\hat{N} = a_T\hat{T} + a_r\hat{N}$$

$$a_T = \frac{dv}{dt}$$

$$a_r = -\frac{v^2}{r}$$

$$|\vec{a}| = \sqrt{a_T^2 + a_r^2}$$

p71

(예제4-6)

$$a_T = 0.3 \ (m/sec^2)$$

$$a_r = -\frac{v^2}{r} = -\frac{36}{500} \ (m/sec^2) = -0.072 \ (m/sec^2)$$

$$a = \sqrt{a_T^2 + a_r^2} = \sqrt{(0.3)^2 + (-0.072)^2} = 0.309 \ (m/sec^2)$$

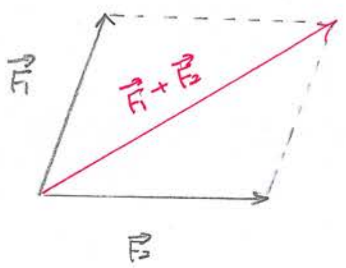
CH5. 운동의 법칙

◎ 힘의 개념 (force)

힘: 물체의 속도를 변화시키는 것

기본힘 { 중력 : 질량과 질량사이의 힘
전자기력: 전하와 전하사이의 힘
약력 :
강력 :

힘: vector　　　　　　　　단위: $N = kg\,\frac{m}{sec^2}$



◎ Newton 제1법칙

Newton 제1법칙 : 관성의 법칙

물체에 외력이 없다면 정지했던 물체는 정지 상태를 유지하고
운동하던 물체는 그 상태를 유지한다.

⇒ 물체의 외력이 없으면 가속도가 0 이다.

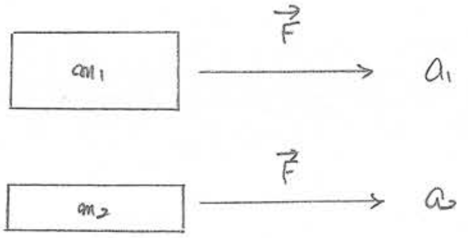
(퀴즈 5.1)

마찰이 없으면　　답 (a)

마찰을 고려하면　　답 (d)　　※

3 질량 (mass)

질량 : 속도의 변화를 거스르는 정도를 나타내는 물체의 속성



if $a_1 > a_2$, $m_1 < m_2$. if $a_1 < a_2$, $m_2 < m_1$.

$$\Rightarrow \quad \boxed{\frac{m_1}{m_2} = \frac{a_2}{a_1}}$$

질량 : scalar 양 , 단위 = kg

무게 = mg

Ex) $W = 60$ kg중

$m = 60$ kg

$W = 60 \times 9.8\ (N) = 588\ (N)$

p84 § Newton 제 2 법칙

$$\boxed{\begin{array}{l}\vec{F}: \text{ 물체가 받는 total 힘} \\ \vec{F} = m\vec{a}\end{array}}$$

Newton 제 2 법칙

$[\vec{F}] = N = kg\ m/sec^2$

$[m] = kg$

$[a] = m/sec^2$

p86

(예제5.1)

$\vec{F} = \vec{F_1} + \vec{F_2}$

$F_x = F_{1x} + F_{2x} = 5\cos 20° + 8\cos 60° = 8.7\ (N)$

$F_y = F_{1y} + F_{2y} = -5\sin 20° + 8\sin 60° = 5.2\ (N)$

$\vec{F} = 8.7\ \hat{x} + 5.2\ \hat{y}\ (N)$

$\vec{a} = \frac{\vec{F}}{m} = \frac{8.7}{0.3}\hat{x} + \frac{5.2}{0.3}\hat{y}\ (m/sec^2)$

$= 29\ \hat{x} + 17\ \hat{y}\ (m/sec^2)$

$a = \sqrt{29^2 + 17^2} = 34\ (m/sec^2)$

$\theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\frac{17}{29} \fallingdotseq 30°$ ※

p87

§ 중력과 무게

(퀴즈 5.4)

$$g_{지구} \cong 100\ g_{달}$$

$$m_1 g_{지구} = m_2 g_{달} = 1\,(N)$$

$$\Rightarrow \frac{m_2}{m_1} = \frac{g_{지구}}{g_{달}} \simeq 100$$

$$m_2 \simeq 100\ m_1$$

(답) b ※.

p88 § 5.6 Newton 제3법칙

Newton 제3법칙 : 작용, 반작용법칙

물체 1이 물체 2에 힘 $\vec{F}_{12}$ 를 주면, 물체 2도 물체 1에 그 크기는 같고, 방향이 반대인 $\vec{F}_{21}$ 을 준다;

$$\vec{F}_{21} = -\vec{F}_{12}$$

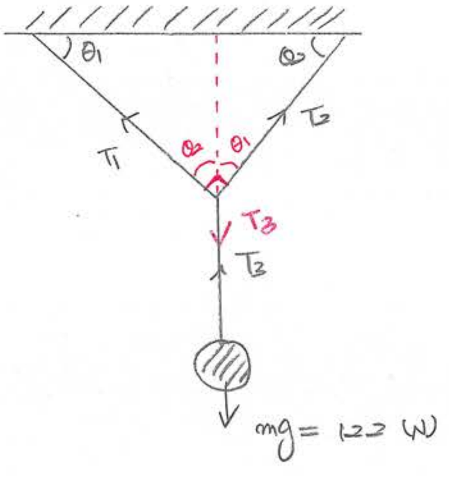
$\vec{F}_{12}$: 작용력

$\vec{F}_{21}$: 반작용력

p90 5 뉴턴법칙의 응용

p92

(예제 5.2)



$\theta_1 = 37^\circ$

$\theta_2 = 53^\circ$

$T_3 = 122$ (N)

$T_2 \sin\theta_1 = T_1 \sin\theta_2$ — ①

$T_1 \cos\theta_2 + T_2 \cos\theta_1 = 122$ — ②

From ①

$$T_2 = T_1 \frac{\sin\theta_2}{\sin\theta_1} \quad — ③$$

③ → ②

$T_1 \cos\theta_2 + T_1 \sin\theta_2 \cot\theta_1 = 122$

$\Rightarrow T_1 \left[\cos\theta_2 + \sin\theta_2 \cot\theta_1 \right] = 122$

$$\Rightarrow T_1 = \frac{122}{\cos\theta_2 + \sin\theta_2 \cot\theta_1} = 73.4 \text{ (N)} \quad — ④$$

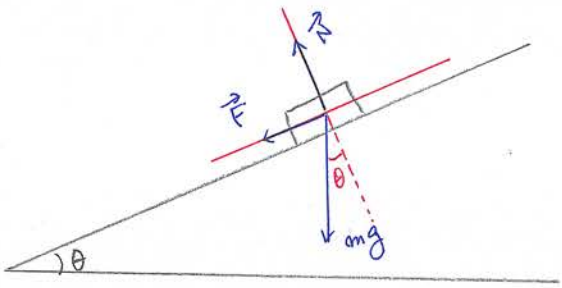
④ → ③

$$T_2 = 73.4 \frac{\sin\theta_2}{\sin\theta_1} = 97.4 \text{ (N)}$$

※

p93

(예제5.3)



(A)

$$N = mg\cos\theta$$

$$F = mg\sin\theta = ma$$

$$a = g\sin\theta$$

(B)

① $v^2 - 0 = 2ad$

$$v = \sqrt{2ad} = \sqrt{2g\sin\theta\, d} = \sqrt{2gd\sin\theta}$$

② $v = 0 + at$

$$t = \frac{v}{a} = \frac{\sqrt{2gd\sin\theta}}{g\sin\theta} = \sqrt{\frac{2d}{g\sin\theta}}$$

p95

(예제 5.4)

(A)

물그기 질량 : m

지구의 중력가속도 : g

(i) 위로 가속도 a를 가질때 $T - mg = ma$

물그기 무게 = 중력 + 관성력 $= mg + ma = m(g+a)$

(ii) 아래로 가속도 a를 가질때 $T - mg = -ma$

물그기 무게 = 중력 + 관성력 $= mg - ma = m(g-a)$

(B) $mg = 40\ (N)$

$a = \pm 2\ (m/sec^2)$

$\Rightarrow m = \frac{40}{g}\ (kg)$

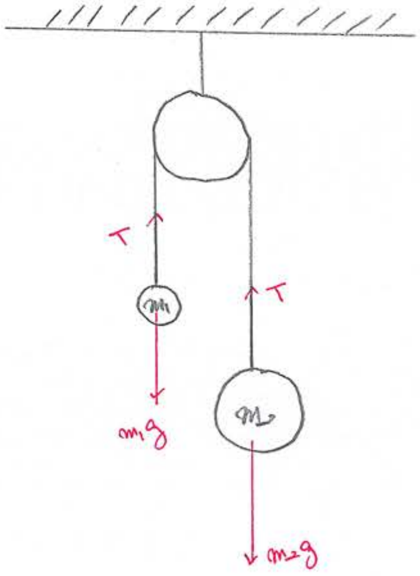
물그기 무게 $= 40\ (N) + ma$

$= 40 + \frac{40}{g}(\pm 2)$

$= 40 \pm \frac{80}{g} = \begin{cases} 48.2\ (N) \\ 31.8\ (N). \end{cases}$

※

(예제 5.5)



$$m_2 g - T = m_2 a \quad — ①$$

$$T - m_1 g = m_1 a \quad — ②$$

② → ①

$$m_2 g - (m_1 g + m_1 a) = m_2 a$$

$$\Rightarrow (m_2 - m_1) g = (m_1 + m_2) a$$

$$\Rightarrow \quad a = \frac{m_2 - m_1}{m_1 + m_2} g \quad — ③$$

③ → ②

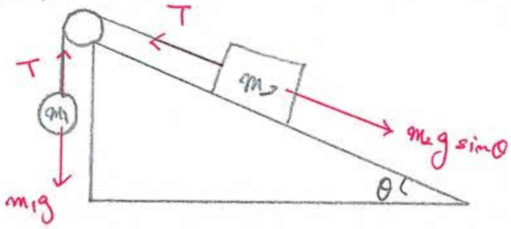
$$T = m_1 g + m_1 a$$

$$= m_1 g \left[1 + \frac{m_2 - m_1}{m_1 + m_2} \right]$$

$$= \frac{2 m_1 m_2}{m_1 + m_2} g \quad ※$$

p97

(예제 5.6)



$$m_2 g \sin\theta - T = m_2 a \quad -①$$

$$T - m_1 g = m_1 a \quad -②$$

② → ①

$$m_2 g \sin\theta - (m_1 a + m_1 g) = m_2 a$$

$$\Rightarrow (m_2 + m_1) a = (m_2 \sin\theta - m_1) g$$

$$\Rightarrow a = \frac{m_2 \sin\theta - m_1}{m_2 + m_1} g \quad -③$$

③ → ②

$$T = m_1 g + m_1 a$$

$$= m_1 g \left(1 + \frac{m_2 \sin\theta - m_1}{m_2 + m_1}\right)$$

$$= \frac{(1+\sin\theta) m_1 m_2}{m_1 + m_2} g.$$

※

p98

§ 마찰력

마찰력 ∝ 수직 항력

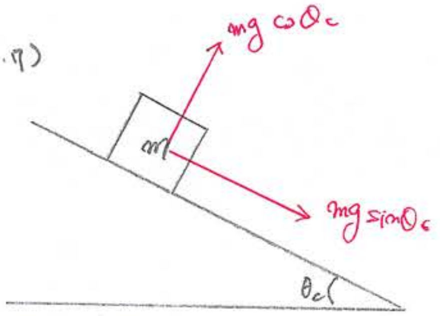
$\vec{f}_s \leq \mu_s \vec{N}$: 정지 마찰력

μ_s: 정지 마찰계수

$\vec{f}_k = \mu_k \vec{N}$: 운동 마찰력

μ_k: 운동 마찰계수

p101
(예제 5.7)

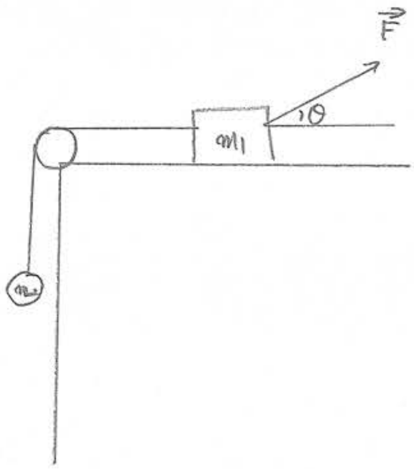


$mg\sin\theta_c = \mu_s\, mg\cos\theta_c$

$\Rightarrow \quad \mu_s = \tan\theta_c$ ※

p102

(예제 5.8)



$$N + F\sin\theta = m_1 g$$

$$\Rightarrow \quad N = m_1 g - F\sin\theta \quad - ①$$

$$F\cos\theta - (\mu_k N + T) = m_1 a \quad - ②$$

$$T - m_2 g = m_2 a \quad - ③$$

③ → ②

$$F\cos\theta - \mu_k N - (m_2 g + m_2 a) = m_1 a$$

$$\Rightarrow (m_1 + m_2) a = F\cos\theta - \mu_k N - m_2 g$$

$$= F\cos\theta - \mu_k (m_1 g - F\sin\theta) - m_2 g$$

$$= F(\cos\theta + \mu_k \sin\theta) - (\mu_k m_1 + m_2) g$$

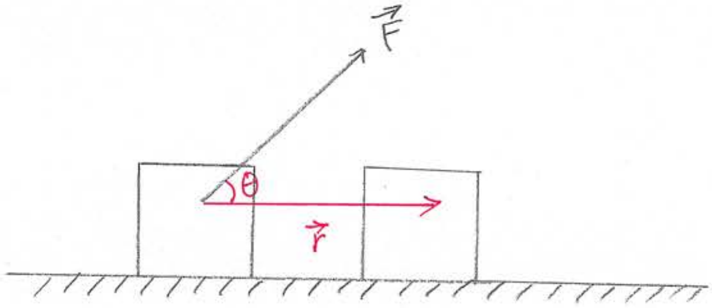
$$\Rightarrow \quad a = \frac{F(\cos\theta + \mu_k \sin\theta) - (\mu_k m_1 + m_2) g}{m_1 + m_2}$$

※.

CH7 계의 에너지

p127

○ 일정한 힘이 한 일



W : $\vec{F}$가 물체에 행한 일

$$W \equiv F r \cos\theta$$

$$[W] = [F][r] = Nm \equiv J \text{ (joule)}$$

(Ex) $1\,J = 1\,Nm = 1\,(kg\,m^2/sec^2)$

p129

(퀴즈 7.1) a

(퀴즈 7.2) c − a − d − b

P129

(예제 7.1)

$$W = 50\,N \times 3\,m \times \cos 30^\circ$$

$$= 150\ \frac{\sqrt{3}}{2}\ (J)$$

$$= 75\sqrt{3}\ (J)$$

$$\simeq 130\ (J)$$

#

p120

② 두 벡터의 스칼라 곱

$$\boxed{\begin{array}{l} \vec{A}\cdot\vec{B} \equiv |\vec{A}|\,|\vec{B}|\cos\theta \\ \theta:\ \vec{A}\text{와 }\vec{B}\text{ 사잇각} \end{array}}$$

Properties

(i) $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

(ii) $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$

(iii) $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$

$\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$

$\Rightarrow \boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$

pf) $\vec{A}\cdot\vec{B}$

$= (A_x\hat{x} + A_y\hat{y} + A_z\hat{z})\cdot(B_x\hat{x} + B_y\hat{y} + B_z\hat{z})$

$= A_xB_x \underbrace{\hat{x}\cdot\hat{x}}_{=1} + A_xB_y \underbrace{\hat{x}\cdot\hat{y}}_{=0} + A_xB_z \underbrace{\hat{x}\cdot\hat{z}}_{=0}$

$+ A_yB_x \underbrace{\hat{y}\cdot\hat{x}}_{=0} + A_yB_y \underbrace{\hat{y}\cdot\hat{y}}_{=1} + A_yB_z \underbrace{\hat{y}\cdot\hat{z}}_{=0}$

$+ A_zB_x \underbrace{\hat{z}\cdot\hat{x}}_{=0} + A_zB_y \underbrace{\hat{z}\cdot\hat{y}}_{=0} + A_zB_z \underbrace{\hat{z}\cdot\hat{z}}_{=1}$

$= A_xB_x + A_yB_y + A_zB_z$ ※

(iv) $\vec{A}\cdot\vec{A} = |\vec{A}|^2$

※ 일정한 힘이 한 일

$$\underline{\underline{W = F\,r\cos\theta = \vec{F}\cdot\vec{r}}}$$

p132

(예제 7.2)

$$\vec{A} = 2\hat{x} + 3\hat{y}$$

$$\vec{B} = -\hat{x} + 2\hat{y}$$

(A) $\vec{A}\cdot\vec{B} = 2\times(-1) + 3\times 2 = 4$

(B) $|\vec{A}| = \sqrt{2^2+3^2} = \sqrt{13}$

$|\vec{B}| = \sqrt{(-1)^2+2^2} = \sqrt{5}$

$$\vec{A}\cdot\vec{B} = 4 = |\vec{A}|\,|\vec{B}|\cos\theta$$

$$\cos\theta = \frac{4}{\sqrt{13}\cdot\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{65}} = 60.3^\circ$$

※

p132

(예제 7.3)

$$\Delta\vec{r} = 2\hat{x} + 3\hat{y} \quad (m)$$

$$\vec{F} = 5\hat{x} + 2\hat{y} \quad (N)$$

(A) $|\Delta\vec{r}| = \sqrt{2^2+3^2} = \sqrt{13}\,(m)$

$|\vec{F}| = \sqrt{5^2+2^2} = \sqrt{29}\ (N)$

(B) $W = \vec{F}\cdot\Delta\vec{r} = 5\times 2 + 2\times 3\ (J) = 16\ (J)$ ※.

p133

§7.4 변하는 힘이 한 일

$W = \vec{F} \cdot \vec{r}$: 일정한 힘이 한 일

만약 $\vec{F}$ 가 물체의 위치에 따라 변 한 다면

$$\boxed{W = \int \vec{F} \cdot d\vec{r} = \int (F_x\,dx + F_y\,dy + F_z\,dz)}$$

만약 물체의 변위가 x-방향이라면

$$dy = dz = 0$$

$$\Rightarrow \quad W = \int_{x_i}^{x_f} F_x\,dx$$

p134

(예제 7.4)

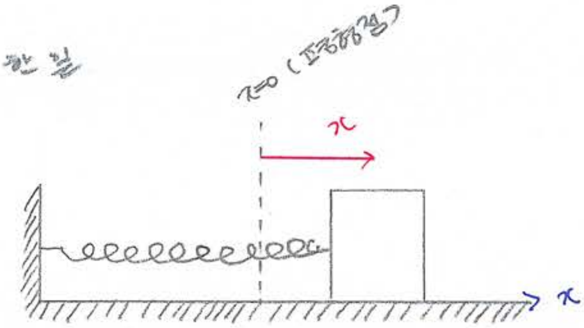
$$W = \int_0^6 F_x\,dx$$

$$= \underbrace{\int_0^4 F_x\,dx}_{20\,(J)} + \underbrace{\int_4^6 F_x\,dx}_{5\,(J)}$$

$$= 25\,(J)$$ ※

p134

* 용수철이 한 일



$\vec{F}$: spring이 물체에 주는 힘

$\vec{F} = -kx\,\hat{x}$: 복원력

k: spring 상수

W: 물체가 x_i에서 x_f로 움직일때 스프링이 물체에 해준 일

$$W = \int_{x_i}^{x_f} (-kx)\,dx$$

$$= -k\frac{1}{2}x^2 \Big|_{x=x_i}^{x=x_f}$$

$$= -\frac{k}{2}\left[x_f^2 - x_i^2\right]$$

$$= \frac{k}{2}x_i^2 - \frac{k}{2}x_f^2$$

<퀴즈 7.4>

$F = kx$: 사람이 물체에 가한 힘

$$W_1 = \int_0^{-x} F\,dx = \frac{k}{2}x^2 \Big|_{x=0}^{x=-x} = \frac{k}{2}x^2$$

$$W_2 = \int_{-x}^{-2x} F\,dx = \frac{k}{2}x^2 \Big|_{x=-x}^{x=-2x} = \frac{k}{2}\left[4x^2 - x^2\right] = \frac{3}{2}kx^2$$

화살 한개 장전을 위해 필요한 일 $= W_1 = \frac{k}{2}x^2$

화살 두개 장전을 위해 필요한 일 $= W_1 + W_2 = 2kx^2$

$\Rightarrow$ 4배 *.

p137

(예제 7.5)

(A)

$$Kd = mg$$

$$K = \frac{mg}{d} = \frac{0.55\,kg \times 9.8\,m/sec^2}{0.02\,m} = 2.7 \times 10^2\ (N/m)$$

(B)

$$W = \frac{K}{2}x_i^2 - \frac{K}{2}x_f^2 \quad -①$$

$$\left.\begin{aligned} x_i &= 0 \\ x_f &= d \end{aligned}\right\} -②$$

$$W = -\frac{K}{2}d^2 = -5.4 \times 10^{-2}\ (J)$$ *

P138

§7.5 일-운동에너지 원리

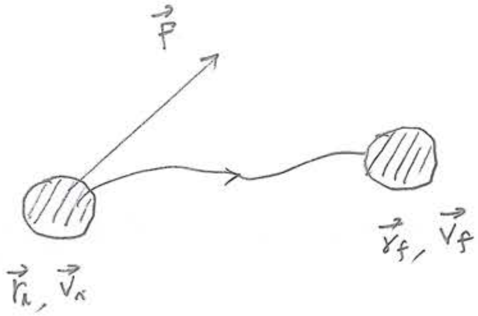
운동에너지 (Kinetic energy)

$$K = \frac{1}{2} m v^2$$

m: 질량

v: 속력

* W: 물체가 받는 total 힘이 한 일



$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$$

$$= \int_{\vec{r}_i}^{\vec{r}_f} m \vec{a} \cdot d\vec{r}$$

$$= \int_{\vec{r}_i}^{\vec{r}_f} m \frac{d\vec{v}}{dt} \cdot d\vec{r}$$

$$= \int_{\vec{v}_i}^{\vec{v}_f} m \frac{d\vec{r}}{dt} \cdot d\vec{v}$$

$$= \int_{v_i}^{v_f} m v \, dv$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$= K_f - K_i$$

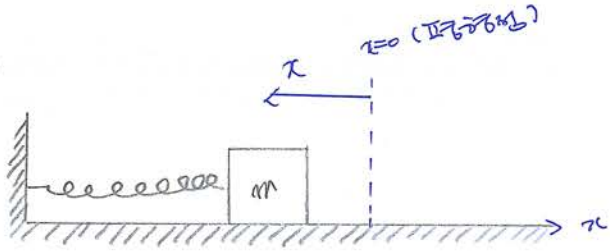
$$W = K_f - K_i$$

W: 물체에 주어진 total 힘이 한 일

K_f: 최종 운동에너지

K_i: 처음 운동에너지

(퀴즈 7.5)



$F = -kx\,\hat{x}$

W : $x = x_i$ 에서 $x = x_f$ 로 갈 때 F가 한 일

$$W = \int_{x_i}^{x_f} (-kx)\,dx = -\frac{k}{2}\left(x_f^2 - x_i^2\right) = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 \quad -①$$

(A) 첫번째 화살이 $x=0$ 을 통과할 때의 속도 : v_1

$x_i = -x, \quad v_i = 0$

$x_f = 0, \quad v_f \equiv v_1$

$$W = \frac{1}{2}kx^2 - 0 = \frac{1}{2}mv_1^2 - 0$$

$$v_1 = \sqrt{\frac{k}{m}}\,x$$

(B) 두번째 화살이 $x=0$ 을 통과할 때의 속도 : v_2

$x_i = -2x, \quad v_i = 0$

$x_f = 0, \quad v_f = v_2$

$$W = \frac{1}{2}k(2x)^2 - 0 = \frac{1}{2}mv_2^2 - 0$$

$$v_2 = \sqrt{\frac{k}{m}}\cdot 2x = 2v_1$$

//

P141

(예제 7.6)

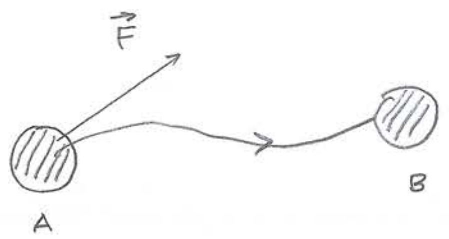
W: $\vec{F}$가 한 일

$W = 12 \times 3 \ (J) = 36 \ (J)$

$W = \frac{1}{2} m v_f^2 - 0$

$v_f = \sqrt{\frac{2W}{m}} = 3.5 \ (m/sec)$

P146 ◎ 보존력과 비보존력



W: $\vec{F}$가 한 일

$$W = \int_A^B \vec{F} \cdot d\vec{r}$$

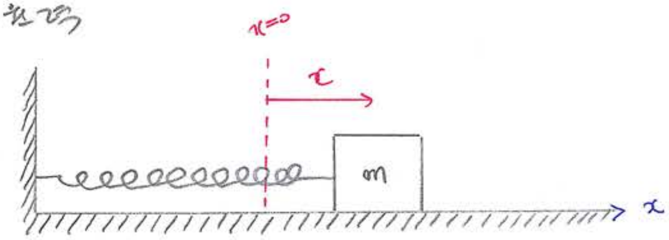
$\vec{F}$: 보존력 (conservative force)

If

① W가 물체의 이동경로와 무관하고 양 끝점에만 의존한다.

② 폐경로를 따라 (즉 A=B) 움직인경우 $W=0$ 이다.

Ex) 복원력



W: 물체가 $x=x_i$ 에서 $x=x_f$ 로 움직일때

복원력 $\vec{F} = -kx\,\hat{x}$ 이 한 일

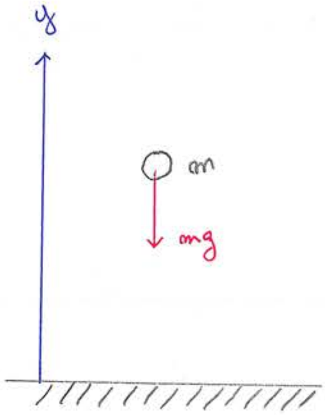
$$W = \int_{x_i}^{x_f} (-kx)\,dx$$

$$= \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

If $x_i = x_f$, $\quad W = 0$

$\Rightarrow \vec{F} = -kx\,\hat{x}$ is conservative force ⁂

Ex) 중력



$\vec{F} = -mg\,\hat{y}$

W : 물체가 y_i 에서 y_f 로 움직일때 중력이 한 일

$$W = \int_{y_i}^{y_f} (-mg)\, dy$$

$$= -mg\,(y_f - y_i)$$

$$= mgy_i - mgy_f$$

If $y_i = y_f$, $W = 0$

중력은 보존력 !!

If $\vec{F}$ is not conservative force,

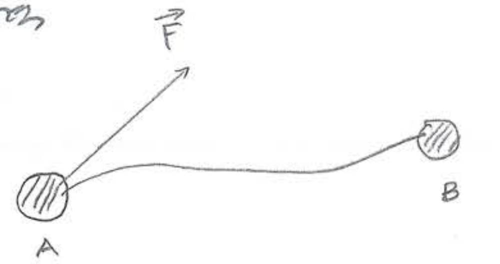
$\vec{F}$: 비보존력 (non-conservative force)

(Ex) 마찰력

p149

8. 보존력과 위치 에너지의 관계

$\vec{F}$: 보존력



× 기준점

$$W = \int_A^B \vec{F}\cdot d\vec{r}$$

$$= \int_A^{기준점} \vec{F}\cdot d\vec{r} + \int_{기준점}^{B} \vec{F}\cdot d\vec{r}$$

$$= \left(-\int_{기준점}^{A} \vec{F}\cdot d\vec{r}\right) - \left(-\int_{기준점}^{B} \vec{F}\cdot d\vec{r}\right)$$

보존력의 위치 에너지 (potential energy)

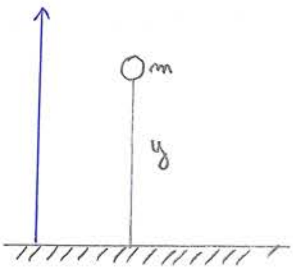
$$\boxed{U(\vec{r}) \equiv -\int_{기준점}^{\vec{r}} \vec{F}\cdot d\vec{r}}$$

$\Rightarrow$ $\underline{\underline{W = U(A) - U(B)}}$

일과 위치 에너지 관계

p140

(Ex) 중력

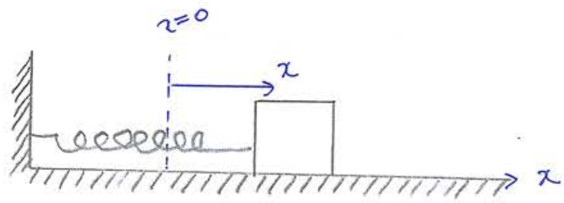


기준점 : $y = 0$ (지표면)

$$U(y) = -\int_0^y (-mg)\,dy = mgy$$ 중력 위치 에너지

p146

(Ex) 복원력



기준점 : $x = 0$

복원력 : $\vec{F} = -kx\,\hat{x}$

$$U(x) = -\int_0^x (-kx)\,dx = \frac{1}{2}kx^2$$ 탄성 위치 에너지

* potential energy 로 부터 보존력 $\vec{F}$ 를 구하는 방법

$$\vec{F} = -\vec{\nabla} U(\vec{r})$$

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$: del operator

Ex) 중력

$$U = mgy$$

$$\vec{F} = -\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)(mgy)$$

$$= -\hat{x}\underbrace{\frac{\partial}{\partial x}(mgy)}_{=0} - \hat{y}\underbrace{\frac{\partial}{\partial y}(mgy)}_{mg} - \hat{z}\underbrace{\frac{\partial}{\partial z}(mgy)}_{=0}$$

$$= -mg\,\hat{y}$$

※

Ex) 복원력

$$U = \frac{1}{2}kx^2$$

$$\vec{F} = -\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\left(\frac{1}{2}kx^2\right)$$

$$= -\hat{x}\frac{\partial}{\partial x}\left(\frac{1}{2}kx^2\right)$$

$$= -\hat{x}(kx)$$

$$= -kx\,\hat{x}$$

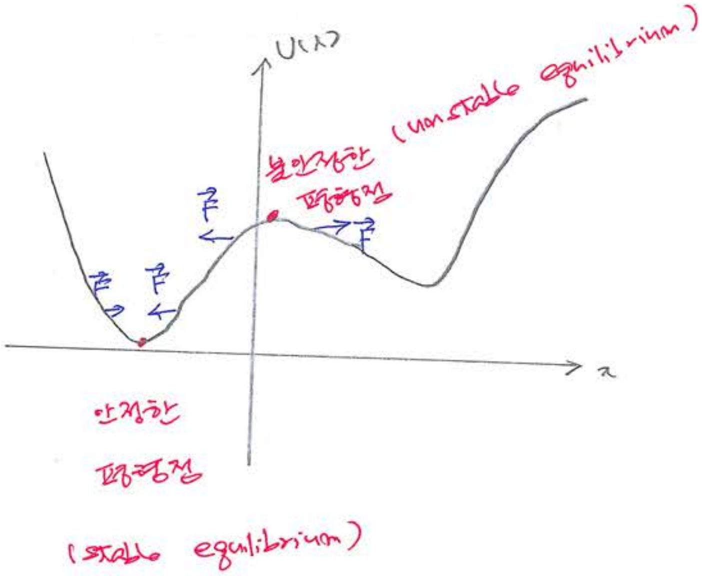
※.

p150

§ 계의 평형

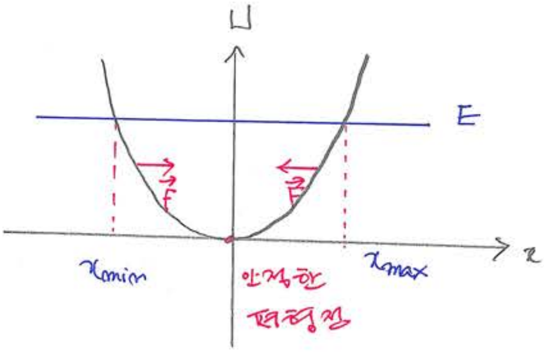
평형점 : $\vec{F}=0$ 인 점

(equilibrium) potential의 기울기가 0인 점



Ex) 탄성에너지

$U=\frac{1}{2}kx^2$



x_{max}, x_{min} : turning point ※.

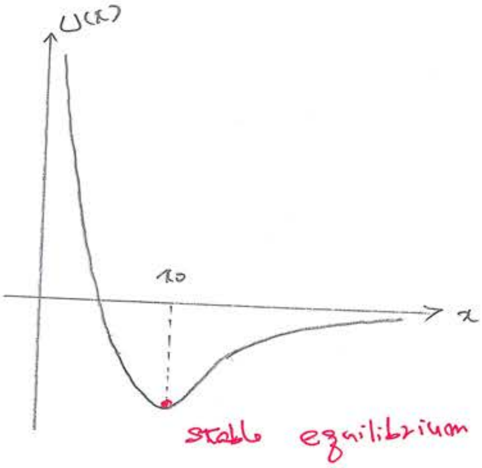
p152

(예제 7.8)

$$U(x) = 4\epsilon\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right]$$

$\epsilon = 1.51 \times 10^{-22}$ (J)

$\sigma = 0.263$ (nm) $= 2.63 \times 10^{-10}$ (m)



$$\frac{dU}{dx} = 4\epsilon\left[-12\sigma^{12}\frac{1}{x^{13}} + 6\sigma^{6}\frac{1}{x^{7}}\right]$$

$$= 24\epsilon\sigma^{6}\frac{1}{x^{7}}\left[1 - \frac{2\sigma^{6}}{x^{6}}\right] = 0$$

$$x_0 = (2\sigma^{6})^{\frac{1}{6}} = 0.295 \text{ (nm)}$$ *

CH 9. 선운동량과 충돌

§ 선운동량과 보존

$\vec{F}_{12}$: 입자 1이 입자 2에 주는 힘

$\vec{F}_{21}$: 입자 2가 입자 1에 주는 힘

Then from Newton's third law

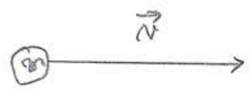
$$\vec{F}_{12} + \vec{F}_{21} = 0$$

$$\Rightarrow m_1 \vec{a}_1 + m_2 \vec{a}_2 = 0$$

$$\Rightarrow \frac{d}{dt}(m_1 \vec{v}_1 + m_2 \vec{v}_2) = 0$$

$$\Rightarrow m_1 \vec{v}_1 + m_2 \vec{v}_2 = \text{const.}$$

* 정의: 선운동량 (linear momentum)



$$\vec{p} \equiv m \vec{v}$$

* $\vec{F} = \frac{d\vec{p}}{dt}$

note) 단위

$$[\vec{p}] = \text{kg m/sec}$$

$$\Rightarrow \vec{p}_1 + \vec{p}_2 = \text{const}$$

운동량 보존법칙

If there is no external force for particle system,

$$\sum_i \vec{P}_i = \text{const}$$

P187

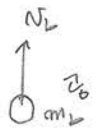
(예제 9.1)

$$60 \times v + 0.5 \times 50 = 0$$

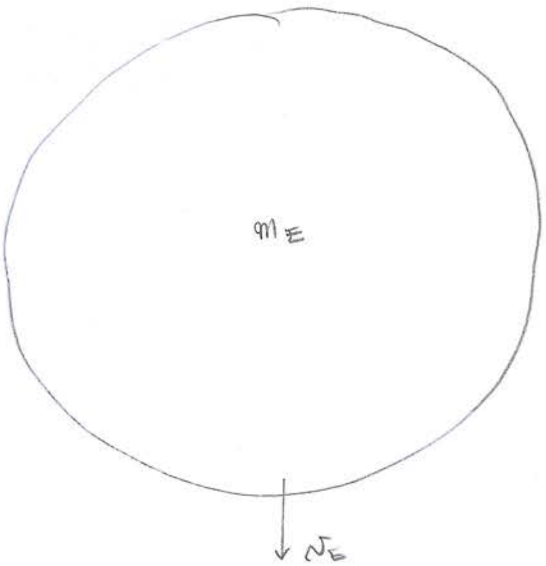
$$v = -\frac{5}{12}\ (\text{m/sec}) = -0.42\ (\text{m/sec})\ ※$$

P188

(예제 9.2)



$$K_B = \frac{1}{2} m_b v_b^2$$



$$K_E = \frac{1}{2} m_E v_E^2$$

$$m_b v_b + m_E v_E = 0$$

$$\Rightarrow \frac{v_E}{v_b} = -\frac{m_b}{m_E}$$

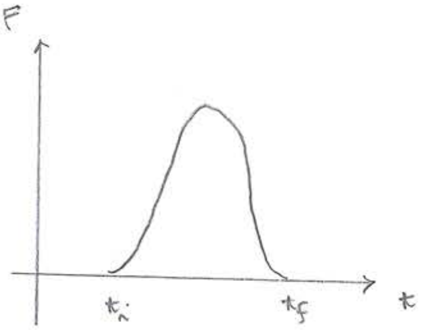
$$\frac{K_E}{K_B} = \frac{\frac{1}{2} m_E v_E^2}{\frac{1}{2} m_b v_b^2} = \left(\frac{m_E}{m_b}\right)\left(\frac{v_E}{v_b}\right)^2 = \frac{m_b}{m_E}$$

$m_b = 1\ kg$

$m_E = 10^{24}\ kg$

$\frac{K_E}{K_B} \sim 10^{-24}$ #

p188 § 충격량과 운동량



$\vec{I} = \int_{t_i}^{t_f} \vec{F}\, dt$: 충격량 $[\vec{I}] = N\,sec = kg\,m/sec = [\vec{p}]$

충격량-운동량 정리 (impulse-momentum theorem)

$$\vec{I} = \int_{t_i}^{t_f} \vec{F}\, dt$$

$$= \int_{t_i}^{t_f} \frac{d\vec{p}}{dt}\, dt$$

$$= \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p}$$

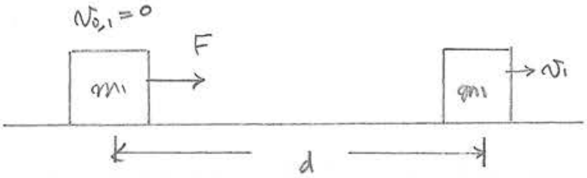
$$= \vec{p}_f - \vec{p}_i$$

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \vec{I}$$

p190

(예제9.3)

(i) $m_1 > m_2$



$$a_1 = \frac{F}{m_1}$$

$$v_1^2 = 2a_1 d = 2\frac{F}{m_1}\cdot d$$

$$\Rightarrow v_1 = \sqrt{\frac{2Fd}{m_1}}$$

$$\Rightarrow P_1 = \Delta P_1 = m_1 v_1 = \sqrt{2m_1 F d} \quad -①$$

* 충격량 : $v_1 = a_1 t_1$

$$t_1 = \frac{v_1}{a_1} = \frac{m_1}{F}\sqrt{\frac{2Fd}{m_1}} = \sqrt{\frac{2m_1 d}{F}}$$

$$I_1 = F t_1 = \sqrt{2m_1 F d} = \Delta P_1$$

$$\Rightarrow K_1 = \frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1 \frac{2Fd}{m_1} = Fd$$

* For m_2

$$P_2 = \sqrt{2m_2 F d}$$

$$K_2 = Fd$$

(답) $P_1 > P_2$, $K_1 = K_2$

(ii) $I = F \Delta t$

$P_1 = P_2$

$m_1 v_1 = m_2 v_2$

$$\frac{K_2}{K_1} = \frac{\frac{P_2^2}{2m_2}}{\frac{P_1^2}{2m_1}} = \frac{m_1}{m_2} > 1$$

$\Rightarrow \quad K_2 > K_1$

(답) $P_1 = P_2, \quad K_2 > K_1$

P191

(예제 9.3)

$\Delta \vec{P} = m\vec{V_f} - m\vec{V_i}$

$= m(\vec{V_f} - \vec{V_i})$

$= m\left[2.6 - (-15)\right] \hat{x}$

$= 1500 \times 17.6 \ \hat{x} \quad (\mathrm{kg\,m/sec})$

$= 2.64 \times 10^4 \ \hat{x} \quad (\mathrm{kg\,m/sec})$

$\vec{I} = \Delta \vec{P} = 2.64 \times 10^4 \ \hat{x} \quad (\mathrm{kg\,m/sec})$

$\vec{I} = \vec{F} \Delta t$

$\vec{F} = \frac{\vec{I}}{\Delta t} = \frac{1}{0.15} \ 2.64 \times 10^4 \ \hat{x} \ (N) = 1.76 \times 10^5 \ \hat{x} \quad (N)$

P192

§ 일차원 충돌

If $m_1, m_2, v_{1,i}$ and $v_{2,i}$ are given, what is $v_{1,f}$ and $v_{2,f}$?

Since there is no external force,

$$m_1 \vec{v}_{1,i} + m_2 \vec{v}_{2,i} = m_1 \vec{v}_{1,f} + m_2 \vec{v}_{2,f}$$

[1] 탄성충돌 (Elastic collision)

If there is no heat loss, we call "elastic collision". In this case

$$\frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2 \quad — ①$$

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f} \quad — ②$$

From ①

$$m_1 (v_{1,i}^2 - v_{1,f}^2) = m_2 (v_{2,f}^2 - v_{2,i}^2)$$

$$\Rightarrow m_1 (v_{1,i} - v_{1,f})(v_{1,i} + v_{1,f}) = m_2 (v_{2,f} - v_{2,i})(v_{2,f} + v_{2,i}) \quad — ③$$

From ②

$$m_1 (v_{1,i} - v_{1,f}) = m_2 (v_{2,f} - v_{2,i}) \quad — ④$$

From ③ and ④

$$v_{1,i} + v_{1,f} = v_{2,f} + v_{2,i}$$

$$\Rightarrow v_{1,f} - v_{2,f} = v_{2,i} - v_{1,i} \quad — ①$$

$$m_1 v_{1,f} + m_2 v_{2,f} = m_1 v_{1,i} + m_2 v_{2,i} \quad — ②$$

$$v_{1,f} = \frac{\begin{vmatrix} v_{2,i} - v_{1,i} & -1 \\ m_1 v_{1,i} + m_2 v_{2,i} & m_2 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ m_1 & m_2 \end{vmatrix}}$$

$$= \frac{1}{m_1 + m_2} \left[m_2 (v_{2,i} - v_{1,i}) + m_1 v_{1,i} + m_2 v_{2,i} \right]$$

$$= \frac{1}{m_1 + m_2} \left[(m_1 - m_2) v_{1,i} + 2 m_2 v_{2,i} \right]$$

$$v_{2,f} = \frac{1}{m_1 + m_2} \begin{vmatrix} 1 & v_{2,i} - v_{1,i} \\ m_1 & m_1 v_{1,i} + m_2 v_{2,i} \end{vmatrix}$$

$$= \frac{1}{m_1 + m_2} \left[m_1 v_{1,i} + m_2 v_{2,i} - m_1 (v_{2,i} - v_{1,i}) \right]$$

$$= \frac{1}{m_1 + m_2} \left[2 m_1 v_{1,i} + (m_2 - m_1) v_{2,i} \right]$$

$$\Rightarrow \boxed{\begin{aligned} v_{1,f} &= \frac{(m_1 - m_2) v_{1,i} + 2 m_2 v_{2,i}}{m_1 + m_2} \\ v_{2,f} &= \frac{2 m_1 v_{1,i} + (m_2 - m_1) v_{2,i}}{m_1 + m_2} \end{aligned}}$$

(Ex1) $m_1 = m_2 \equiv m$

$v_{1,f} = v_{2,i}$ "속도 교환 발생"

$v_{2,f} = v_{1,i}$

(Ex2) $v_{2,i} = 0$

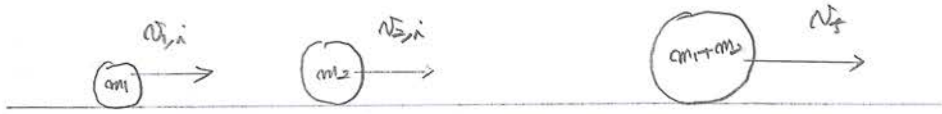
$$v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1,i}$$

$$v_{2,f} = \frac{2m_1}{m_1 + m_2} v_{1,i}$$

If $m_2 \gg m_1$, $v_{1,f} = -v_{1,i}$, $v_{2,f} = 0$ ※

[2] 완전 비탄성 충돌

> 완전 비탄성 충돌
> ⇒ 충돌후 두물체가 한덩어리가 되어 같이 움직이는 충돌



$$m_1 v_{1,i} + m_2 v_{2,i} = (m_1 + m_2) v_f$$

$$\Rightarrow \boxed{v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2}}$$

(퀴즈 9.6)

m_1 : 탁구공 $\quad v_{1,i}$

m_2 : 볼링공 $\quad v_{2,i} = 0$

$$v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1,i}$$

$$v_{2,f} = \frac{2m_1}{m_1 + m_2} v_{1,i}$$

$$P_{1,f} = m_1 v_{1,f} = \frac{m_1(m_1 - m_2)}{m_1 + m_2} v_{1,i}$$

$$|P_{1,f}| = \frac{m_1(m_2 - m_1)}{m_1 + m_2} v_{1,i}$$

$$P_{2,f} = m_2 v_{2,f} = \frac{2m_1 m_2}{m_1 + m_2} v_{1,i}$$

$$P_{2,f} - |P_{1,f}| = \frac{m_1(m_2 + m_1)}{m_1 + m_2} v_{1,i} > 0$$

$$\Rightarrow \quad |P_{1,f}| < P_{2,f}$$

$$K_{1,f} = \frac{1}{2} m_1 v_{1,f}^2 = \frac{1}{2} m_1 \frac{(m_1 - m_2)^2}{(m_1 + m_2)^2} v_{1,i}^2$$

$$K_{2,f} = \frac{1}{2} m_2 v_{2,f}^2 = \frac{1}{2} m_2 \frac{4m_1^2}{(m_1 + m_2)^2} v_{1,i}^2$$

$$\frac{K_{1,f}}{K_{2,f}} = \frac{1}{4}\left(\frac{m_2}{m_1}\right)\left(1 - \frac{m_1}{m_2}\right)^2 \gg 1$$

$$K_{1,f} \gg K_{2,f}$$

(답) b

(예제 9.4)

충돌 전 : $P_i = mv$

$K_i = \frac{1}{2}mv^2$

충돌 후 : $P_f = m\left(\frac{v}{2}\right) + m\left(\frac{v}{2}\right) = mv$ ok

$K_f = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$

$K_f \neq K_i$

탄성충돌 에서는 불가능 !!

(문제) 공 4와 5를 붙인다면 ?

$P_i = m v_{1,i}$

$P_f = m v_{1,f} + 2m v_{4,5}$

$\Rightarrow \quad m v_{1,i} = m v_{1,f} + 2m v_{4,5} \quad - ①$

$K_i = \frac{1}{2} m v_{1,i}^2$

$K_f = \frac{1}{2} m v_{1,f}^2 + \frac{1}{2}(2m) v_{4,5}^2$

$\Rightarrow \quad K_i = K_f$

$\frac{1}{2} m v_{1,i}^2 = \frac{1}{2} m v_{1,f}^2 + m v_{4,5}^2 \quad - ②$

From ① and ②

$$\boxed{\begin{aligned} v_{4,5} &= \frac{2}{3} v_{1,i} \\ v_{1,f} &= -\frac{1}{3} v_{1,i} \end{aligned}}$$

※

p196

(예제9.5)

m_1: 소형차

m_2: 대형차

$$m_1 v_{1,i} = (m_1+m_2) v_f$$

$$v_f = \frac{m_1}{m_1+m_2} v_{1,i} = \frac{20}{3} \ (m/sec)$$ ※

P197

(예제9.6)

$$m_1 v_{iA} = (m_1+m_2) v_f$$

$$v_f = \frac{m_1}{m_1+m_2} v_{1,A} \quad -①$$

에너지 보존 법칙

$$\frac{1}{2}(m_1+m_2) v_f^2 = (m_1+m_2) g h$$

$$v_f = \sqrt{2gh} \quad -②$$

② → ①

$$v_{1,A} = \frac{m_1+m_2}{m_1} \sqrt{2gh}$$ ※

p198

§ 2차원 충돌

$$m_1 v_{1i,x} + m_2 v_{2i,x} = m_1 v_{1f,x} + m_2 v_{2f,x}$$

$$m_1 v_{1i,y} + m_2 v_{2i,y} = m_1 v_{1f,y} + m_2 v_{2f,y}$$

탄성: $\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$

$\Rightarrow$ 식 3개, unknown 변수 4개 ($v_{1f,x}$, $v_{1f,y}$, $v_{2f,x}$, $v_{2f,y}$)

$\Rightarrow$ 일반적으로는 풀수 없다

※ 만약 $v_{1f,x}$, $v_{1f,y}$, $v_{2f,x}$, $v_{2f,y}$ 중 1개의 정보를 알려주면 풀수 있다 !!

* 완전 비탄성 충돌에서는 $\vec{V}_f$ 를 구할수 있다;

$$m_1 \vec{V}_{1,i} + m_2 \vec{V}_{2,i} = (m_1 + m_2) \vec{V}_f$$

$$\vec{V}_f = \frac{m_1 \vec{V}_{1,i} + m_2 \vec{V}_{2,i}}{m_1 + m_2}$$

p199

(예제 9.7)

$m_1 = 1500\,(kg)$ $\vec{V}_{1,i} = 25\,\hat{x}\ (m/sec)$

$m_2 = 2500\,(kg)$, $\vec{V}_{2,i} = 20\,\hat{y}\ (m/sec)$

$$\vec{V}_f = \frac{1}{4000}\left[1500 \times 25\,\hat{x} + 2500 \times 20\,\hat{y}\right]$$

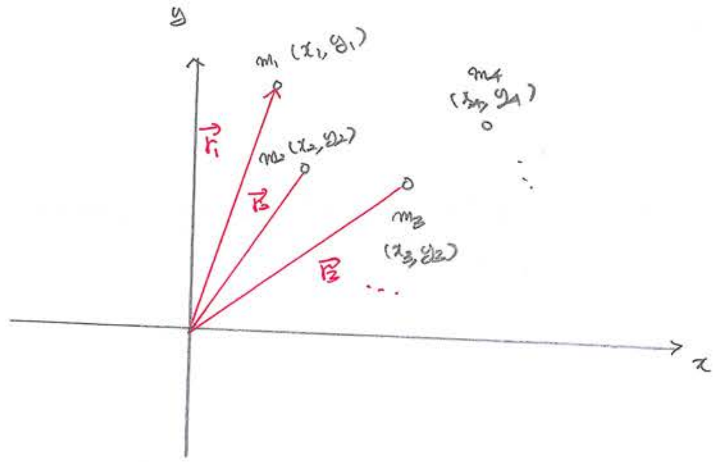
$$= \frac{75}{8}\,\hat{x} + \frac{25}{2}\,\hat{y} \quad (m/sec)$$

$$v_f = \sqrt{\left(\frac{75}{8}\right)^2 + \left(\frac{25}{2}\right)^2} = 15.6\ (m/sec)$$

$$\theta = \tan^{-1}\left(\frac{\frac{25}{2}}{\frac{75}{8}}\right) = 53.1^\circ$$

※

§ 질량 중심 (center of mass)



$$x_{cM} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{1}{M} \sum_i m_i x_i$$

$$y_{cM} = \frac{1}{M} \sum_i m_i y_i$$

$$M = m_1 + m_2 + \cdots = \sum_i m_i$$

(x_{cM}, y_{cM}) : center of mass

$$\vec{r}_{cM} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

For continuum body

$$\vec{r}_{cM} = \frac{1}{M} \int \vec{r}\, dm$$

p203

(예제9.8)

m_1 (1kg) (1, 0)

m_2 (1kg) (2, 0)

m_3 (2kg) (0, 2)

$$x_{CM} = \frac{1}{m_1+m_2+m_3}(m_1x_1 + m_2x_2 + m_3x_3)$$

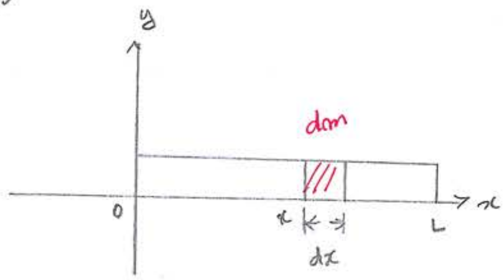
$$= \frac{1}{4}(1+2) = \frac{3}{4} \text{ (m)}$$

$$y_{CM} = \frac{1}{4}(m_1y_1 + m_2y_2 + m_3y_3) = \frac{4}{4} = 1 \text{ (m)}$$

$$\vec{r}_{CM} = \frac{3}{4}\hat{x} + 1\hat{y} \text{ (m)}$$ ※

(예제9.9)

(A)



$$x_{CM} = \frac{1}{M}\int x\,dm \quad — ①$$

$$L : M = dx : dm$$

$$dm = \frac{M}{L}dx \quad — ②$$

$$x_{cm} = \frac{1}{M}\int_0^L x \ \frac{M}{L}\,dx$$

$$= \frac{1}{L}\int_0^L x\,dx$$

$$= \frac{1}{L}\cdot\frac{L^2}{2}$$

$$= \frac{L}{2}$$

(B) $\lambda = \alpha x$: 단위 길이당 질량

$$M = \int dm \qquad -①$$

$$1 : \alpha x = dx : dm$$

$$\Rightarrow dm = \alpha x\,dx \qquad -②$$

② → ①

$$M = \int_0^L \alpha x\,dx = \alpha\frac{L^2}{2} \qquad -③$$

$$x_{cm} = \frac{1}{M}\int x\,dm$$

$$= \frac{1}{M}\int_0^L x\cdot\alpha x\,dx$$

$$= \frac{\alpha}{M}\int_0^L x^2\,dx$$

$$= \frac{\alpha}{M}\cdot\frac{1}{3}L^3$$

$$= \frac{\alpha}{3}L^3 \ \frac{2}{\alpha L^2}$$

$$= \frac{2}{3}L \qquad ※.$$

§ 입자계의 운동

$$\vec{r}_{CM} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} \qquad \text{Center of mass}$$

$$\vec{V}_{CM} \equiv \frac{d\vec{r}_{CM}}{dt} = \frac{\sum_i m_i \vec{V}_i}{\sum_i m_i} \iff M\vec{V}_{CM} = \sum_i m_i \vec{V}_i = \sum_i \vec{P}_i = \vec{P}_{tot}$$

① 입자계의 total 선운동량은 입자계의 모든 질량이 CM에 있는 경우 CM의 선운동량과 일치한다.

② 외력이 존재하더라도 CM에 위치한 관찰자에게는 $\vec{P}_{tot} = 0$ 으로 항상 보존한다.

$$\vec{a}_{CM} \equiv \frac{d\vec{V}_{CM}}{dt} = \frac{\sum_i m_i \vec{a}_i}{\sum_i m_i} \iff M\vec{a}_{CM} = \sum_i m_i \vec{a}_i = \vec{F}_{ext}$$

입자계의 CM은 마치 그곳에 모든 질량이 있는것처럼 외력에 대하여 행동한다.

$$\text{If } \vec{F}_{ext} = 0, \quad \vec{a}_{CM} \equiv \frac{d\vec{V}_{CM}}{dt} = 0$$

$\Rightarrow \vec{V}_{CM}$ is constant vector

$\Rightarrow M\vec{V}_{CM}$ is constant vector

$\Rightarrow \underline{\vec{P}_{tot} = \sum_i \vec{P}_i = \text{constant vector}}$ 선운동량 보존법칙

p.206

(예제 9.10)

$$\vec{F}_{ext} = 0$$

$$\vec{P}_i = M\vec{V}_i = M\,(300\ m/sec)\,\hat{y}$$

$$\vec{P}_f = \frac{M}{3}\,(450\ m/sec)\,\hat{y} + \frac{M}{3}\,(240\ m/sec)\,\hat{x} + \frac{M}{3}\,\vec{V}$$

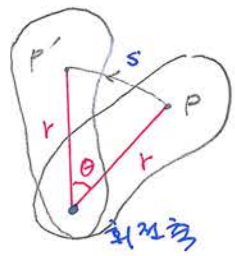
$$\vec{P}_i = \vec{P}_f \Rightarrow \vec{V} = (-240\,\hat{x} + 450\,\hat{y})\ (m/sec)$$ //

CH.10 강체의 회전

강체 (rigid body) : ◦변형이 없는 물체

◦ 입자들 간의 상대위치가 변하지 않는 물체

※ 각위치, 각속도, 각 가속도



$s = r\theta$

$\theta = \frac{s}{r}$: radian 각도

환산

$\pi = 180^\circ$

Ex) ① 30°

$\pi : 180 = x : 30$

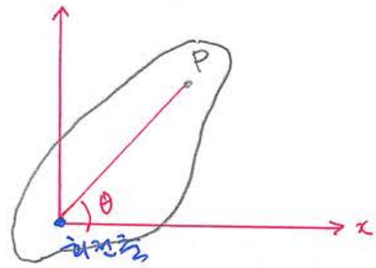
$\Rightarrow \frac{\pi}{6}$

② $\frac{2}{3}\pi$

$\pi : 180 = \frac{2}{3}\pi : x$

$x = 120^\circ$

※



θ_i : 시간 t_i 에서의 θ

θ_f : 시간 t_f 에서의 θ

$\Delta\theta \equiv \theta_f - \theta_i$: 각변위 (angle displacement) $\Leftrightarrow$ $\Delta x = x_f - x_i$

$\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i}$: 평균 각속도 $\Leftrightarrow$ $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$

$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$: 순간 각속도 $\Leftrightarrow$ $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

$\alpha_{avg} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$: 평균 각가속도 $\Leftrightarrow$ $a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$

$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$: 순간 각가속도 $\Leftrightarrow$ $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

$[\omega] = rad/sec$, $[\alpha] = rad/sec^2$

* 강체가 고정축에 대하여 회전할때 물체위의 모든 입자는 모두 같은 각변위, 각속도, 각가속도를 갖는다.

p216

회전운동학

*등각속도 원운동 (ω = 일정, $\alpha = 0$) ⇔ (v = 일정) 등속도 운동

$$\theta_f = \theta_i + \omega t \quad \Leftrightarrow \quad x_f = x_i + vt$$

*등각가속도 원운동 (α = 일정) ⇔ (a = 일정) 등가속도 운동

$$\omega_f = \omega_i + \alpha t \quad \Leftrightarrow \quad v_f = v_i + at$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \quad \Leftrightarrow \quad x_f = x_i + v_i t + \frac{1}{2}at^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \quad \Leftrightarrow \quad v_f^2 = v_i^2 + 2a(x_f - x_i)$$

p218

(예제 10.1)

$\alpha = 3.5$ (rad/sec^2)

(A) $t_i = 0$, $\omega_i = 2$ (rad/sec)

$$\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2 = 2 \times 2 + \frac{1}{2} \times 3.5 \times 4 = 11\,(\text{rad}) = 620^\circ$$

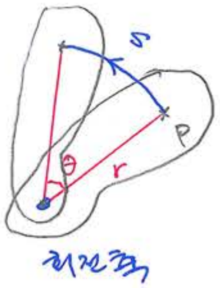
(B) $\dfrac{\theta_f - \theta_i}{2\pi} = \dfrac{11}{2\pi} = 1.75$ (회전) $= 1.75$ (rev)

rev = revolution (회전수)

(C) $\omega_f = \omega_i + \alpha t = 2 + 3.5 \times 2 = 9$ (rad/sec) ※

p218

◎ 회전운동과 선운동의 물리량



$$s = r\theta$$

$$v = \frac{ds}{dt}$$: 접선속도

$$v = \frac{ds}{dt} = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt} = r\omega$$ 접선속도

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha$$ 접선가속도

$$a_c = \frac{v^2}{r} = r\omega^2$$: 구심가속도

* 강체의 모든 점에서 ω와 α가 일정하지만, 접선속도 v, 접선가속도 a_t, 구심가속도 a_c는 일정하지 않다 !!

$$a = \sqrt{a_t^2 + a_c^2} = r\sqrt{\alpha^2 + \omega^4}$$

(예제 10.2)

(A)

$$\omega_i = \frac{v}{r_i} = \frac{1.3\ (m/sec)}{2.3\times 10^{-2}\ (m)} = 57\ (rad/sec)$$

$$= 57\times 60\ (rad/min)$$

$$= \frac{57\times 60}{2\pi}\ (rev/min)$$

$$= 5.4\times 10^{2}\ (rev/min)$$

$$\omega_f = \frac{v}{r_f} = \frac{1.3\ (m/sec)}{5.8\times 10^{-2}\ (m)} = 22\ (rad/sec)$$

$$= 22\times 60\ (rad/min)$$

$$= \frac{22\times 60}{2\pi}\ (rev/min)$$

$$= 2.1\times 10^{2}\ (rev/min)$$

(C) $\omega_f = \omega_i + \alpha t$

$\omega_f = 22\ (rad/sec)$

$\omega_i = 57\ (rad/sec)$

$t = 74\times 60 + 33 = 4473\ (sec)$

$\Rightarrow \alpha = -7.8\times 10^{-3}\ (rad/sec^2)$

(B) $\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$

$$= 22\times 4473 + \frac{1}{2}(-7.8\times 10^{-3})\times 4473^2$$

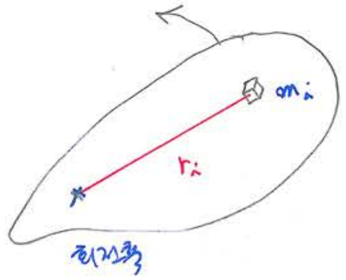
$$= 1.8\times 10^{5}\ (rad)$$

$$= \frac{1.8\times 10^{5}}{2\pi}\ (rev)$$

$$= 2.8\times 10^{4}\ (rev)$$

※.

p.221 § 회전운동에너지



K_i : m_i의 운동에너지

$K_i = \frac{1}{2} m_i v_i^2$ (v_i : 접선속도)

$\Rightarrow K_i = \frac{1}{2} m_i (r_i \omega)^2$

$= \frac{1}{2} m_i r_i^2 \omega^2$ — ①

강체의 회전운동에너지

$K = \sum_i K_i = \frac{1}{2} \left(\sum_i m_i r_i^2 \right) \omega^2$ — ②

$I \equiv \sum_i m_i r_i^2$: 관성 moment (moment of inertia) — ③

* r_i : 질량 m_i와 회전축 사이의 거리

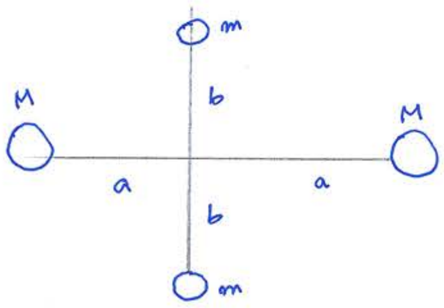
$[I] = kg\,m^2$

Then

$K = \frac{1}{2} I \omega^2 \quad \leftrightarrow \quad K = \frac{1}{2} m v^2$

p222

(예제 10.3)



(A) 회전축 = y축

$$I = \sum_i m_i r_i^2$$

$$= 2Ma^2$$

$$K = \frac{1}{2} I\omega^2 = Ma^2\omega^2$$

(B) 회전축 = z축

$$I = \sum_i m_i r_i^2$$

$$= 2mb^2 + 2Ma^2$$

$$K = \frac{1}{2} I\omega^2 = (mb^2 + Ma^2)\omega^2$$ ※

p223

δ 관성 모멘트 계산

$$I = \sum_i m_i r_i^2 = \int r^2 dm$$

r_i, r: 회전축과 질량소 사이의 거리

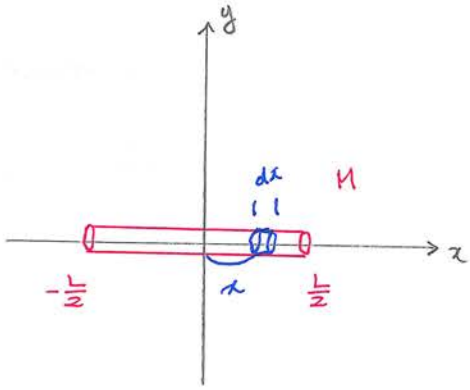
만약 물체의 밀도가 ρ 라면

$\Rightarrow \rho = \frac{M}{V}$

$\Rightarrow dm$의 부피 $= \rho\, dV$

$$I = \int \rho r^2 dV$$

p225
(예제 10.4)



$I = \int r^2 dm$ — ①

$r = x$ — ②

$L : M = dx : dm$

$\Rightarrow dm = \frac{M}{L} dx$ — ③

②, ③ → ①

$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \frac{M}{L} dx$$

$$I = \frac{M}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} x^2\,dx$$

$$= \frac{2M}{L}\int_{0}^{\frac{L}{2}} x^2\,dx$$

$$= \frac{2M}{L}\cdot\frac{1}{3}x^3\Big|_{x=0}^{x=\frac{L}{2}}$$

$$= \frac{2M}{3L}\cdot\frac{L^3}{8}$$

$$= \frac{1}{12}ML^2$$

* 만약 회전축이 막대 끝이 나면 ?

$$r = \frac{L}{2} + x$$

$$\Rightarrow I = \int_{-\frac{L}{2}}^{\frac{L}{2}} \left(\frac{L}{2}+x\right)^2 \frac{M}{L}\,dx$$

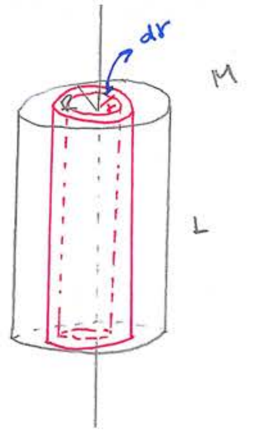
$\frac{L}{2} + x = y \quad (dx = dy)$

$$= \int_0^L y^2 \frac{M}{L}\,dy$$

$$= \frac{M}{L}\,\frac{1}{2}\,y^3\Big|_{y=0}^{y=L}$$

$$= \frac{1}{3}ML^2 \quad *$$

(예제 10.6)



$$I = \int r^2 \, dm \quad -①$$

$$r = r \quad -②$$

$$\pi R^2 L : M = 2\pi r \, dr \, L : dm$$

$$\Rightarrow \quad dm = \frac{2\cancel{\pi} r \, dr \cancel{L}}{\cancel{\pi} R^2 \cancel{L}} M = \frac{2M}{R^2} r \, dr \quad -③$$

②, ③ → ①

$$I = \int_0^R r^2 \frac{2M}{R^2} r \, dr$$

$$= \frac{2M}{R^2} \int_0^R r^3 \, dr$$

$$= \frac{2M}{R^2} \cdot \frac{1}{4} R^4$$

$$= \frac{1}{2} M R^2$$

※

p224

(퀴즈 10.4) ω: 각속력

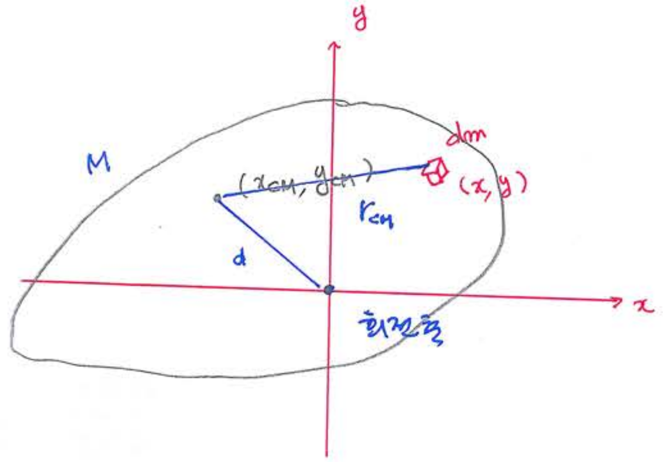
속이 빈 실린더: $K = \frac{1}{2} I \omega^2 = \frac{1}{2} M R^2 \omega^2$

속이 찬 실린더: $K = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \left(\frac{1}{2} M R^2\right) \omega^2 = \frac{1}{4} M R^2 \omega^2$

(답) a

\#

(평행축의 원리)



Put

$$\left.\begin{aligned} x &= x_{cM} + x' \\ y &= y_{cM} + y' \end{aligned}\right\} \quad -①$$

$$I = \int r^2 \, dm$$

$$= \int (x^2 + y^2) \, dm$$

$$= \int (x_{cM}^2 + y_{cM}^2 + x'^2 + y'^2 + 2x_{cM}x' + 2y_{cM}y') \, dm$$

$$= \int (x'^2 + y'^2) \, dm + (x_{cM}^2 + y_{cM}^2) \int dm + 2x_{cM} \int x' \, dm + 2y_{cM} \int y' \, dm \quad -②$$

$$x'^2 + y'^2 = r_{CM}^2$$

$$\Rightarrow \int (x'^2 + y'^2)\, dm$$

$$= \int r_{CM}^2\, dm$$

$$= I_{CM} \qquad — ③$$

$$x_{CM}^2 + y_{CM}^2 = d^2$$

$$\Rightarrow (x_{CM}^2 + y_{CM}^2) \int dm = Md^2 \qquad — ④$$

$$\int x'\, dm$$

$$= \int (x - x_{CM})\, dm$$

$$= \int x\, dm - x_{CM} \int dm \qquad \left(\Leftarrow x_{CM} = \frac{\int x\, dm}{\int dm} \right)$$

$$= \int x\, dm - \frac{\int x\, dm}{\int dm} \int dm$$

$$= \int x\, dm - \int x\, dm$$

$$= 0 \qquad — ⑤$$

By same way

$$\int y'\, dm = 0 \qquad — ⑥$$

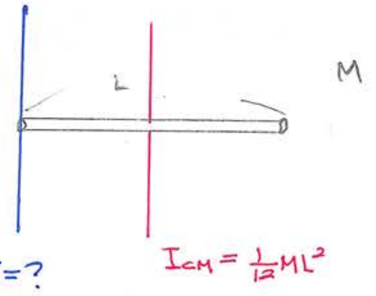
③, ④, ⑤, ⑥ → ②

$$\underline{\underline{I = I_{CM} + Md^2}}$$

평행축의 정리

P227

(예제 10.6)

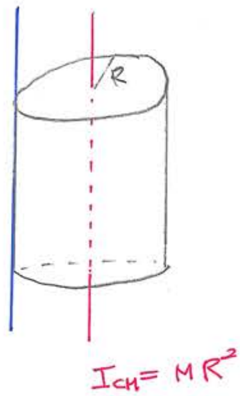


$I = I_{CM} + Md^2 \qquad (d = \frac{L}{2})$

$= \frac{1}{12}ML^2 + \frac{1}{4}ML^2$

$= \frac{1}{3}ML^2$

(ex) 속이 빈 실린더



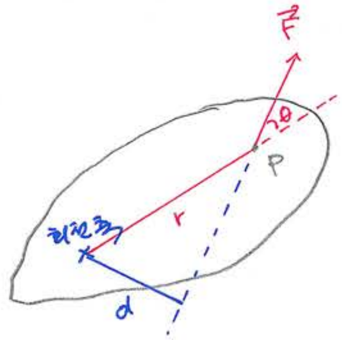
$I = I_{CM} + Md^2 \qquad (d = R)$

$= MR^2 + MR^2$

$= 2MR^2$

※

◎ 토크 (torque)



τ: 회전력

$$\tau = F r \sin\theta = F d$$ (d: moment arm)

If force are F_i, $\tau = \sum_i F_i d_i$.

$\Rightarrow$ $\tau_{total} = 0$ $\Rightarrow$ 회전운동에서 평형상태 (단 반시계방향 $\tau > 0$ / 시계방향 $\tau < 0$)

$\Updownarrow$

($F = 0$ $\Rightarrow$ 병진운동에서 평형상태)

p229

(예제 10.7)

(A) T_2: 반시계 방향 $\tau_2 = R_2 T_2$

T_1: 시계 방향 $\tau_1 = -R_1 T_1$

$$\tau_{total} = \tau_1 + \tau_2 = R_2 T_2 - R_1 T_1$$

(B) $\tau_{total} = 0.5 \times 15 - 1 \times 5 = 2.5$ (Nm)

반시계 방향 !!

p229

ⓢ 알짜 torque 를 받는 강체

$$\tau_{total} = \sum_i F_i r_i \sin\theta_i$$

$$= \sum_i r_i (F_i)_{접선}$$

$$= \sum_i r_i m_i a_i \qquad (a_i : 접선가속도)$$

$$= \sum_i r_i m_i r_i \alpha \qquad (a_i = \alpha r_i)$$

$$= \left(\sum_i m_i r_i^2\right) \alpha$$

$$= I\alpha$$

$$\Rightarrow \quad \tau = I\alpha \qquad \leftrightarrow \qquad F = ma$$

p231

(퀴즈 10.6)

(i) $\alpha = \frac{\tau}{I}$ $(\alpha < 0)$, $\omega = \omega_i$, $\omega_f = 0$

$$0 = \omega_i + \alpha \Delta t$$

$$\Delta t = \frac{\omega_i}{-\alpha} = -\frac{I\omega_i}{\tau}$$

(ii) $\alpha = \frac{\tau}{2I}$ $(\alpha < 0)$, $\omega = \omega_i$, $\omega_f = 0$

$$0 = \omega_i + \frac{\tau}{2I} t$$

$$t = -\frac{\omega_i}{\frac{\tau}{2I}} = -\frac{2I\omega_i}{\tau_i} = 2\Delta t$$

(답) b

p231

(예제 10.8)

$$I = \frac{1}{3}ML^2$$

$$\tau = \frac{L}{2}Mg$$

$$\Rightarrow \alpha = \frac{\tau}{I} = \frac{\frac{L}{2}Mg}{\frac{1}{3}ML^2} = \frac{3g}{2L}$$

$$a_t = L\alpha = \frac{3g}{2}$$ ※

p232

(예제 10.10)

$$\alpha = \frac{\tau}{I} = \frac{TR}{I} \quad - ①$$

$$mg - T = ma$$

$$\Rightarrow a = \frac{mg - T}{m} \quad - ②$$

$$a = R\alpha \quad - ③$$

From ①, ②, ③

$$T = \frac{mg}{1 + \frac{mR^2}{I}}$$

$$a = \frac{g}{1 + \frac{I}{mR^2}}$$

$$\alpha = \frac{a}{R} = \frac{g}{R + \frac{I}{mR}}$$

if $I = 0$,

$T = 0$, $a = g$

if $I = \infty$,

$T = mg$, $a = 0$

§ 벡터 곱 (외적)

$$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta = A_xB_x + A_yB_y + A_zB_z \quad : \text{내적}$$

* 외적

$$\vec{A}\times\vec{B} = \vec{C}$$

$$|\vec{C}| = |\vec{A}|\,|\vec{B}|\sin\theta$$

$\vec{C}$의 방향 : 오른손 법칙

① $\vec{A}\times\vec{A} = 0$

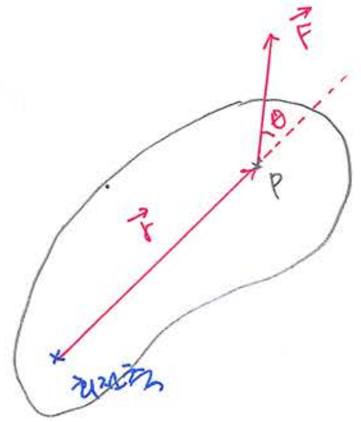
② $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

③ If $\vec{A}\times\vec{B} = 0$, $\vec{A} \parallel \vec{B}$ (if $\vec{A}\cdot\vec{B} = 0$, $\vec{A}\perp\vec{B}$)

④ $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$

⑤ at coordinate

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\hat{x} + (A_zB_x - A_xB_z)\hat{y} + (A_xB_y - A_yB_x)\hat{z}$$



$$\tau = rF\sin\theta$$

$$\Rightarrow \underline{\vec{\tau} = \vec{r}\times\vec{F}}$$

$\vec{r}$: 회전축으로부터 힘이 미치는 곳까지의 위치 vector

~~p250~~

(예제 11.1)

$$\vec{A} = 2\hat{x} + 3\hat{y}$$

$$\vec{B} = -\hat{x} + 2\hat{y}$$

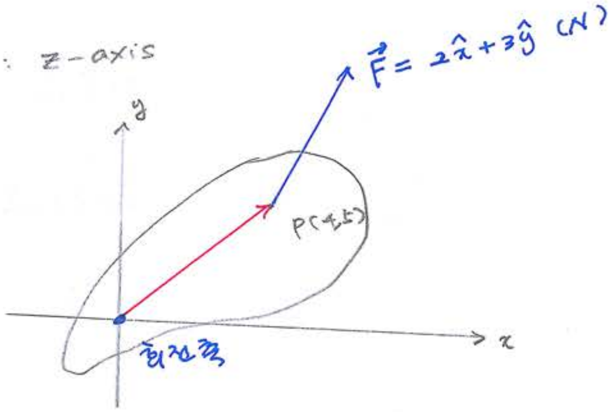
$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 3 & 0 \\ -1 & 2 & 0 \end{vmatrix} = 7\hat{z}$$

$$\vec{B}\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -1 & 2 & 0 \\ 2 & 3 & 0 \end{vmatrix} = -7\hat{z}$$

$$\Rightarrow \vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

(예제 11.2)

회전축 : z-axis



$$\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 4 & 5 & 0 \\ 2 & 3 & 0 \end{vmatrix} = 2\hat{z} \ (Nm)$$

※

§ 각운동량

회전	병진
θ	x
$\omega = \frac{d\theta}{dt}$	$v = \frac{dx}{dt}$
$\alpha = \frac{d\omega}{dt}$	$a = \frac{dv}{dt}$
$\alpha = \text{const}$ (등각가속도 운동)	$a = \text{const}$ (등가속도 운동)
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$I = \sum_i m_i r_i^2 = \int r^2\, dm$ r_i, r: 회전축으로부터 미소질량 dm 까지의 거리	m
$\vec{\tau} = I\vec{\alpha} = \vec{r} \times \vec{F}$	$\vec{F} = m\vec{a}$
?	$\vec{p} = m\vec{v}$
?	$\vec{F} = \frac{d\vec{p}}{dt}$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \frac{d}{dt}(\vec{r} \times \vec{p}) - \frac{d\vec{r}}{dt} \times \vec{p} \qquad -①$$

$$\frac{d\vec{r}}{dt} \times \vec{p}$$

$$= \vec{v} \times (m\vec{v})$$

$$= m\,\vec{v} \times \vec{v}$$

$$= 0 \qquad - ②$$

② → ①

$$\vec{\tau} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

$$\vec{L} = \vec{r} \times \vec{p}$$ 각운동량 (angular momentum)

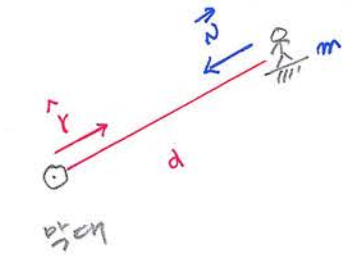
$$[L] = kg\,m^2/sec$$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$ torque 와 각운동량 사이의 관계 ↔ $\vec{F} = \frac{d\vec{p}}{dt}$

p252

(퀴즈 11.2)

(i)

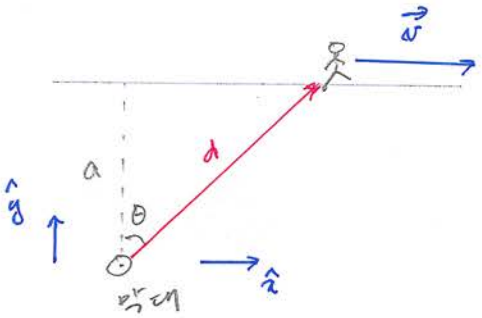


$$\vec r = d\,\hat r$$

$$\vec p = -mv\,\hat r$$

$$\vec L = \vec r \times \vec p = -mvd\;\hat r \times \hat r = 0$$

(ii)



$$\vec r = d\sin\theta\,\hat x + d\cos\theta\,\hat y$$

$$= \sqrt{d^2-a^2}\,\hat x + a\,\hat y$$

$$\vec p = mv\,\hat x$$

$$\vec L = \vec r \times \vec p = \begin{vmatrix} \hat x & \hat y & \hat z \\ \sqrt{d^2-a^2} & a & 0 \\ mv & 0 & 0 \end{vmatrix} = \hat z\,(-mva)$$

$$|\vec L| = mva$$

p252

(예제 11.3)

$$\vec{L} = \vec{r} \times \vec{p} = m\,\vec{r} \times \vec{v}$$

$$|\vec{L}| = m r v \sin\theta \qquad \left(\theta = \frac{\pi}{2}\right)$$

$$= m r v$$

방향: 면 밖의 방향

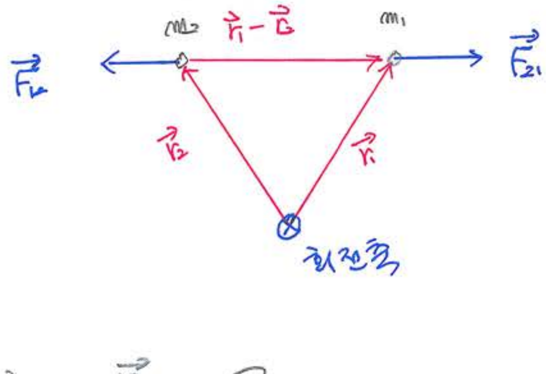
*

* 입자계에서의 각운동량

$$\vec{L} = \sum_i \vec{L}_i = \sum_i \vec{r}_i \times \vec{p}_i$$

$$\vec{\tau}_{total} = \frac{d\vec{L}}{dt} = \sum_i \frac{d\vec{L}_i}{dt} \qquad -①$$

여기서 내력이 만들어주는 torque의 합은 zero 이다.



$$\vec{F}_{12} = -\vec{F}_{21} \qquad -②$$

$$\vec{\tau} = \vec{r}_1 \times \vec{F}_{21} + \vec{r}_2 \times \vec{F}_{12}$$

$$= (\vec{r}_1 - \vec{r}_2) \times \vec{F}_{21}$$

$$= 0 \qquad -③$$

Eq. ① becomes

$$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt} \quad - ④$$

$\vec{\tau}_{ext}$: 외력에 의한 total torque

p258

$\Rightarrow$ 고립계에서 각운동량 보존

고립계 : $\vec{F}_{ext} = 0$, $\vec{\tau}_{ext} = 0$

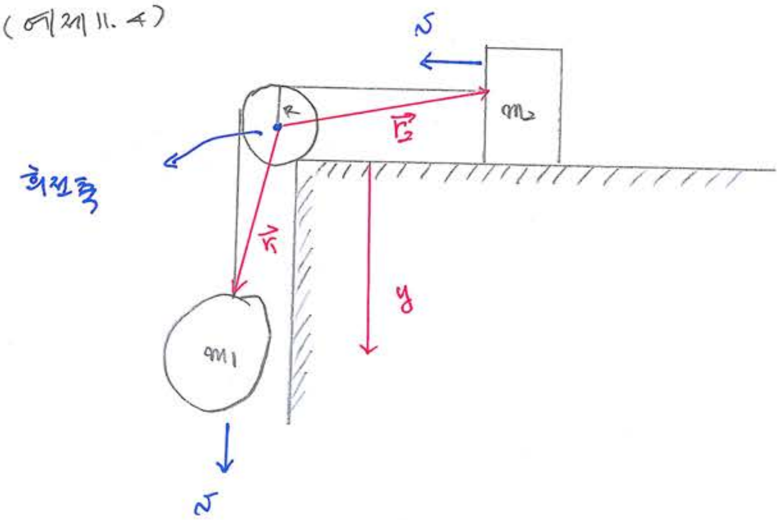
$$\frac{d\vec{L}}{dt} = 0$$

$\vec{L} = \text{const}$ $\Leftrightarrow$ if $\vec{F}_{ext} = 0$, $\vec{P} = \text{const}$

각운동량 보존법칙 선운동량 보존법칙

p254

(예제 11.4)



(i) $\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$ 이용

$L = L_1 + L_2 + L_{도르래}$ — ①

$$\left.\begin{aligned} L_1 &= R m_1 v \\ L_2 &= R m_2 v \\ L_{도르래} &= I\omega = MR^2\left(\frac{v}{R}\right) = MRv \end{aligned}\right\} \text{— ②}$$

② → ①

$L = Rv(m_1 + m_2 + M)$ — ③

$F_{ext} = m_1 g$

$\tau_{ext} = R m_1 g$ — ④

$\tau_{ext} = \frac{dL}{dt}$ — ⑤

③, ④ → ⑤

$$R m_1 g = \frac{d}{dt}\left[Rv(m_1 + m_2 + M)\right] = R(m_1 + m_2 + M)a$$

$$\Rightarrow \quad a = \frac{m_1}{m_1 + m_2 + M} g$$

(ii) 에너지 보존법칙.

$$\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2 - m_1gy = \text{const} \quad -①$$

$$\left.\begin{array}{l} I = MR^2 \\ \omega = \dfrac{v}{R} \end{array}\right\} -②$$

② → ①

$$\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}Mv^2 - m_1gy = \text{const} \quad -③$$

$\frac{d}{dt}$ Eq. ③

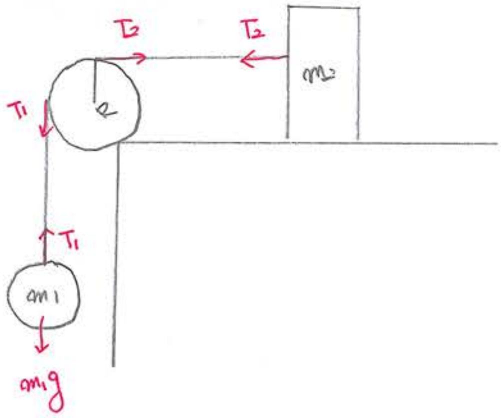
$$(m_1v + m_2v + Mv)\frac{dv}{dt} = m_1g\frac{dy}{dt} \quad -④$$

$$\left.\begin{array}{l} v = \dfrac{dy}{dt} \\ a = \dfrac{dv}{dt} \end{array}\right\} -⑤$$

From ④ and ⑤

$$a = \frac{m_1}{m_1 + m_2 + M}g$$

(iii) elementary method



장력 : m 2종

$$m_1 g - T_1 = m_1 a \quad -①$$

$$T_2 = m_2 a \quad -②$$

$$\tau_{바퀴} = R(T_1 - T_2) = I\alpha = MR^2 \frac{a}{R} = MRa \quad -③$$

From ①, ②, ③

$$a = \frac{m_1}{m_1 + m_2 + M} g$$

※

p259

(퀴즈 11.4)

$$L = I\omega = const$$

돌고난 몸: I가 작아진다 → ω가 커진다.

$$\text{회전운동에너지} = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

돌고난 몸: I가 작아지고 L은 일정 ⇒ 증가한다.

p260

(예제 11.7)

$$L_i = I_{원반}\,\omega_i + I_{사람}\,\omega_i = (I_{원반} + I_{사람})\,\omega_i \quad -①$$

$$\left.\begin{aligned} I_{원반} &= \frac{1}{2}MR^2 \\ I_{사람} &= mR^2 \end{aligned}\right\} -②$$

(p.224 표 10.2)

② → ①

$$L_i = \left(\frac{M}{2} + m\right)R^2\,\omega_i \quad -③$$

$$L_f = (I_{원반} + I_{사람})\,\omega_f \quad -④$$

$$\left.\begin{aligned} I_{원반} &= \frac{1}{2}MR^2 \\ I_{사람} &= mr^2 \end{aligned}\right\} -⑤$$

⑤ → ④

$$L_f = \left(\frac{1}{2}MR^2 + mr^2\right)\omega_f \quad -⑥$$

From $L_i = L_f$,

$$\omega_f = \frac{\left(\frac{M}{2} + m\right)R^2}{\frac{1}{2}MR^2 + mr^2}\,\omega_i \quad \Leftarrow \quad \left(\begin{aligned} &M = 100\,kg, \quad m = 60\,kg \\ &R = 2m, \quad r = 0.5\,m \\ &\omega_i = 2.0\,(rad/sec) \end{aligned}\right)$$

$$= 4.1\,(rad/sec)$$

※

p255

§ 강체의 각운동량

$$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i$$

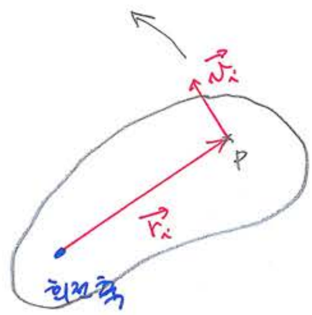
$$= \sum_i \vec{r}_i \times m_i \vec{v}_i$$

$$= \sum_i m_i r_i v_i$$

$$= \sum_i m_i r_i \, r_i \omega \qquad (v_i = r_i \omega)$$

$$= \left(\sum_i m_i r_i^2\right) \omega$$

$$= I \omega$$



$\vec{r}_i \perp \vec{v}_i$

$$\vec{L} = I \vec{\omega} \quad \leftrightarrow \quad \vec{p} = m\vec{v}$$

방향: 오른나사의 법칙

p256

(퀴즈 11.3)

속이 빈 구: $I_1 = \frac{2}{3} MR^2$ (p224 표 10.2)

속이 찬 구: $I_2 = \frac{2}{5} MR^2$

$I_1 > I_2$

※

p256

(예제 11.5)

$$I = \frac{2}{5} M R^2$$

$$\omega = 10 \text{ rev/sec} = 10 \times 2\pi \text{ (rad/sec)} = 20\pi \text{ (rev/sec)}$$

$$L = I\omega = \frac{2}{5} M R^2 \times 20\pi \quad (\text{kg m}^2/\text{sec}^2)$$

if $M = 7\,\text{kg}$, $R = 0.12\,\text{m}$,

$$L = 2.53 \ (\text{kg m}^2/\text{sec})$$

p257

(예제 11.6)

(A) $L = L_{아령} + L_{딸} + L_{시소}$ — ①

$$L_{아령} = I_{아령}\,\omega = \left(m_f \left(\frac{l}{2}\right)^2\right)\omega = \frac{l^2 m_f}{4}\omega$$

$$L_{딸} = I_{딸}\,\omega = \left(m_d \left(\frac{l}{2}\right)^2\right)\omega = \frac{l^2 m_d}{4}\omega$$

— ②

$$L_{시소} = I_{시소}\,\omega = \left(\frac{1}{12} M l^2\right)\omega$$

(p224 표 10.2)

② → ①

$$L = \frac{l^2 \omega}{4}\left[m_f + m_d + \frac{M}{3}\right]$$

(B) $\tau_{ext} = \frac{dL}{dt}$ — ③

$$\tau_{ext} = \frac{l}{2}\cos\theta\, m_f g - \frac{l}{2}\cos\theta\, m_d g = \frac{l}{2}\cos\theta\, g\,(m_f - m_d) \quad — ④$$

④ → ③

$$\frac{l}{2}\cos\theta\, g\,(m_f - m_d) = \frac{l^2}{4}\left(m_f + m_d + \frac{M}{3}\right)\alpha$$

$$\alpha = \frac{2g\cos\theta\,(m_f - m_d)}{l\left(m_f + m_d + \frac{M}{3}\right)}$$

※

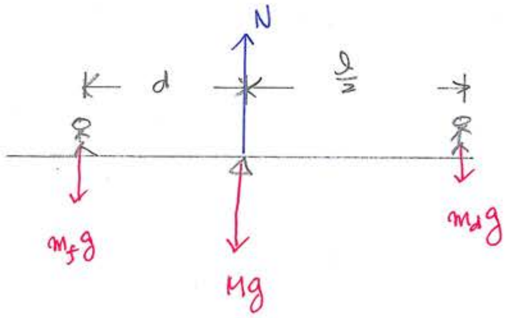
강체의 평형조건

$$\vec{F}_{tot} = \sum \vec{F}_i = 0$$

$$\vec{\tau}_{tot} = \sum_i \vec{\tau}_i = 0$$

p272

(예제12.1)



(A) $\sum_i \vec{F}_i = 0$: $N = (M + m_f + m_d) g$

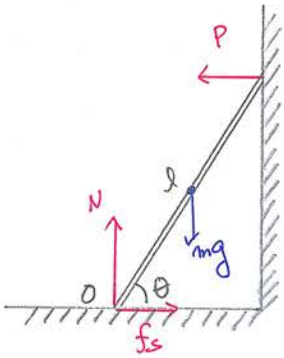
(B) $\sum_i \vec{\tau}_i = 0$

$$d\, m_f g = \frac{l}{2} m_d g$$

$$d = \left(\frac{m_d}{m_f}\right) \frac{l}{2}$$

※

(예제12.2)



$$\sum_i \vec{F_i} = 0$$

$$N = mg \quad -①$$

$$P = f_s \quad -②$$

f_s: 마찰력

$$f_s = \mu_s N = \mu_s mg \quad -③$$

$$\sum_i \vec{\tau_i} = 0 \quad \text{around } O;$$

$$l P \sin(\pi - \theta) = \frac{l}{2} mg \sin\left(\frac{\pi}{2} + \theta\right)$$

$$\cancel{l} P \sin\theta = \frac{\cancel{l}}{2} mg \cos\theta$$

$$\mu_s \cancel{mg} \sin\theta = \frac{\cancel{mg}}{2} \cos\theta$$

$$\tan\theta = \frac{1}{2\mu_s} = \frac{1}{0.8} = 1.25$$

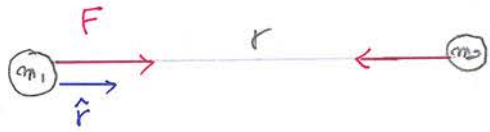
$$\theta = \tan^{-1}(1.25) = 51^\circ$$

//

CH13 만유 인력

p286

§ Newton의 만유인력법칙



$$\boxed{\begin{aligned} F &= G\frac{m_1 m_2}{r^2} \\ G &= 6.673 \times 10^{-11}\ (Nm^2/kg^2) \end{aligned}}$$

Newton의 만유인력법칙

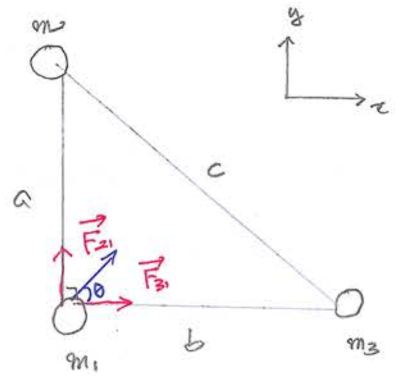
$\vec{F}_{12}$: m_1이 m_2에 주는 힘

$\vec{F}_{21}$: m_2가 m_1에 주는 힘

$$\boxed{\begin{aligned} \vec{F}_{12} &= -G\frac{m_1 m_2}{r^2}\hat{r} \\ \vec{F}_{21} &= G\frac{m_1 m_2}{r^2}\hat{r} \end{aligned}}$$

Vector로 표현한 만유인력법칙

(예제13.1)



$m_1 = m_2 = m_3 = 0.3\,(kg)$

$a = 0.4\,m$

$b = 0.3\,m$

$c = 0.5\,m$

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{21} \quad -①$$

$$\left.\begin{aligned} \vec{F}_{21} &= G\frac{m^2}{a^2}\hat{y} \\ \vec{F}_{31} &= G\frac{m^2}{b^2}\hat{x} \end{aligned}\right\} -②$$

② → ①

$$\vec{F}_1 = Gm^2\left(\frac{1}{b^2}\hat{x} + \frac{1}{a^2}\hat{y}\right)$$

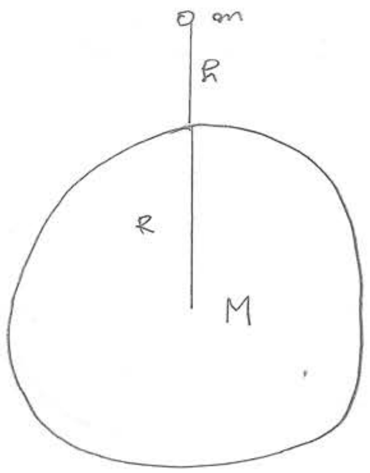
$$= 6.67\times10^{-11}\,\hat{x} + 3.75\times10^{-11}\,\hat{y} \quad (N)$$

$$|\vec{F}_1| = \sqrt{(6.67\times10^{-11})^2 + (3.75\times10^{-11})^2} = 7.65\times10^{-11}\,(N)$$

$$\theta = \tan^{-1}\frac{3.75}{6.67} = \tan^{-1}(0.560) = 29.3^\circ \quad ※$$

p288

ⓔ 중력가속도



$$F_g = G\frac{Mm}{(R+h)^2} \equiv mg$$

$$\Rightarrow \quad g = G\frac{M}{(R+h)^2}$$

중력가속도

* 높이가 높을수록 중력가속도는 작아진다 !!

Ex) 지구

$R \simeq 6.37 \times 10^6$ m

$M = 5.98 \times 10^{24}$ kg

if $h=0$, $g = G\frac{M}{R^2} = 9.8\ (m/sec^2)$

p289

(예제13.2)

$$g = \frac{GM}{(R+h)^2} = 8.83\ (m/sec^2)$$

$$m \cdot 9.8 = 4.22 \times 10^6\ (N)$$

$$m = \frac{4.22 \times 10^6}{9.8}\ (kg)$$

$$W = mg = \frac{4.22 \times 10^6}{9.8} \times 8.83 = 3.80 \times 10^6\ (N) \quad ※$$

p290

(예제13.3)

$$G\frac{Mm}{R^2} = mg$$

$$\Rightarrow g = G\frac{M}{R^2} \quad — ①$$

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3}{4\pi}\frac{1}{R^3}\frac{gR^2}{G} = \frac{3g}{4\pi G R}$$

$$g = 9.8\ (m/sec^2)$$

$$G = 6.673 \times 10^{-11}\ (Nm^2/kg^2)$$

$$R = 6.37 \times 10^6\ (m)$$

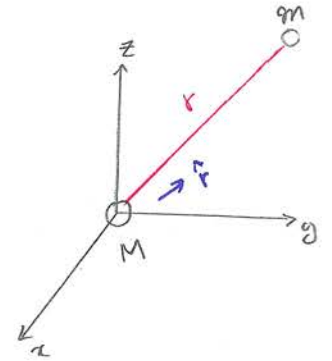
$$\rho = 5.51 \times 10^3\ (kg/m^3) \quad ※$$

§ 중력 위치 에너지

보존력: conservative force $\quad (\vec{\nabla}\times\vec{F}=0)$

$$E=\frac{1}{2}mv^2+V(r)=\text{const}$$

$(\vec{F}=-G\frac{Mm}{r^2}\hat{r})$



$$V(r)=-\int_{\text{기준점}}^{r}\vec{F}\cdot d\vec{r}$$

$$=+\int_{\infty}^{r}G\frac{Mm}{r^2}\,dr$$

$$=-G\frac{Mm}{r}$$

$$\underline{V(r)=-G\frac{Mm}{r}}$$ 중력위치에너지

$$\Rightarrow \quad \frac{1}{2}mv^2-G\frac{Mm}{r}=E=\text{const}$$

* 탈출속도

$$\frac{1}{2}mv^2-G\frac{Mm}{R}=0$$

$v=\sqrt{\frac{2GM}{R}}$: 지상에서 탈출할수 있는 속도

(Ex) 지구

$G=6.673\times10^{-11}\ (Nm^2/kg^2)$

$R=6.37\times10^{6}\ (m)$

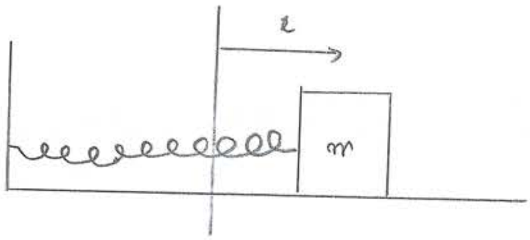
$M=5.98\times10^{24}\ (kg)$

$v_e=11.2\times10^{3}\ (m/sec)$ ※

CH15 진동 (Oscillatory motion)

p334

⑤ 용수철에 연결된 물체의 운동



$F = -kx$ (k: 용수철 상수)

⑤ 단조화 운동

$F = -kx = ma$

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x \quad -①$$

put

$$\omega = \sqrt{\frac{k}{m}} \quad -②$$ (angular frequency, 각 진동수)

② → ①

$$\frac{d^2x}{dt^2} = -\omega^2 x \quad -③$$

$$x = A\cos(\omega t + \phi)$$ A: 진폭 (Amplitude)

주기: $T = \frac{2\pi}{\omega}$

진동수: $f = \frac{1}{T} = \frac{\omega}{2\pi}$

$$v = \frac{dx}{dt} = -A\omega \sin(\omega t + \phi)$$

$$a = \frac{dv}{dt} = -A\omega^2 \cos(\omega t + \phi)$$

p340

(예제 15.1)

$m = 0.2$ (kg) , $K = 5$ (N/m) , $x(t=0) = 0.05$ m , $v(t=0) = 0$

(a)
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5}{0.2}} = 5 \text{ (rad/sec)}$$

$$T = \frac{2\pi}{\omega} = 1.26 \text{ (sec)}$$

(b) $x = A\cos(\omega t + \phi)$ — ①

$v = -\omega A \sin(\omega t + \phi)$ — ②

put $t = 0$ in Eq. ① and ② :

$x(t=0) = A\cos\phi = 0.05$ m

$v(t=0) = -\omega A \sin\phi = 0$

$\Rightarrow$ $\phi = 0$, $A = 0.05$ m — ③

③ ⇒ ①

$x = A\cos\omega t$

$v = -\omega A \sin\omega t$ — ④

$a = -\omega^2 A \cos\omega t$

$A = 0.05$ m

$v_{max} = \omega A = 5 \times 0.05 = 0.25$ (m/sec)

(c) $a_{max} = \omega^2 A = 25 \times 0.05 = 1.25$ (m/sec^2)

(d) $x(t) = 0.05 \cos 5t$ (m)

$v(t) = -0.25 \sin 5t$ (m/sec)

$a(t) = -1.25 \cos 5t$ (m/sec^2)

p341

※ 단조화 진동자의 에너지

$x = A \cos(\omega t + \phi)$

$v = -A\omega \sin(\omega t + \phi)$

$\Rightarrow K = \frac{1}{2} m v^2 = \frac{m}{2} A^2 \omega^2 \sin^2(\omega t + \phi) = \frac{1}{2} k A^2 \sin^2(\omega t + \phi)$

$V = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$

$\underline{\underline{E = K + V = \frac{1}{2} k A^2 = \text{const}}}$

P343

(예제 15.2)

$m = 0.5\ kg$, $K = 20\ (N/m)$, $A = 0.03\ (m)$

(a) $E = \frac{1}{2}KA^2 = \frac{1}{2} \times 20 \times 0.03^2 = 9 \times 10^{-3}\ (J)$

$\frac{1}{2}KA^2 = \frac{1}{2}m v_{max}^2$

$v_{max} = \sqrt{\frac{K}{m}}\,A = 0.19\ (m/sec)$

(b) $E = \frac{1}{2}K(0.02)^2 + \frac{1}{2}mv^2$

$\frac{1}{2}mv^2 = E - \frac{1}{2}K(0.02)^2$

$= \frac{1}{2}K\left[(0.03)^2 - (0.02)^2\right]$

$v = \pm\sqrt{\frac{K}{m}}\ \sqrt{(0.03)^2 - (0.02)^2} = \pm 0.141\ (m/sec)$

(c) $V = \frac{1}{2}K(0.02)^2 = \frac{1}{2} \times 20 \times (0.02)^2 = 4 \times 10^{-3}\ (J)$

$K = E - V = 5 \times 10^{-3}\ (J)$ ※

CH16 파동 (wave)

§ 파동의 종류

횡파: 매질이 파동의 진행방향과 수직으로 움직이는 파동

종파: 매질이 파동의 운동방향과 같은 방향으로 움직이는 파동

§ 파동 방정식 과 간단한 solution

p377

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$: 파동방정식 ─ ①

v: 파동의 속도

solution

$$y = A\sin(kx - \omega t + \phi)$$ ─ ②

A: Amplitude (진폭)

$k = \frac{2\pi}{\lambda}$: wave number (파수)

λ: wave length (파장)

$\omega = \frac{2\pi}{T}$: angular frequency (각진동수)

$T = \frac{2\pi}{\omega} = \frac{1}{f}$: period (주기)

$f = \frac{1}{T} = \frac{\omega}{2\pi}$: frequency (진동수)

ϕ: phase constant (위상 상수)

② → ①

$v = \frac{\omega}{k} = \lambda f$: 파동의 속도 계산 방법

p367

(예제 16.2)

$y = A \sin(kx - \omega t + \phi)$

$A = 0.15\ (m)$, $\lambda = 0.4\ (m)$, $f = 8\ (Hz)$

(A) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.4} = 15.7\ (rad/m)$

$T = \frac{1}{f} = \frac{1}{8}\ (sec) = 0.125\ (sec)$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.125} = 50.3\ (rad/sec)$

$v = \frac{\omega}{k} = \frac{50.3}{15.7} = 3.2\ (m/sec)$

(B) $y = A \sin(kx - \omega t + \phi)$

at $t = 0$ and $x = 0$,

$y = A \sin\phi = 0.15\ (m)$

$\sin\phi = \frac{0.15}{A} = 1$

$\phi = \frac{\pi}{2}$

$y = (0.15\ (m)) \sin\left[(15.7)x - (50.3)t + \frac{\pi}{2}\right]$ 파동함수

※